MATH 5510 – Numerical Analysis October 5, 2016

Assignment 4

1. Consider Gaussian integration on the interval $[-1, 1]$ and $\omega(x) \equiv 1$ and the Legendre polynomials

$$p_k(x) = \frac{k!}{(2k)!} \frac{d^k}{dx^k}(x^2 - 1)^k, \quad k = 0, 1, \ldots.$$

   (a) Show that the leading coefficient of $p_k(x)$ is 1.

   (b) Show that $\int_{-1}^{1} x^j p_k(x) dx = 0$, for $j < k$.

   (c) Show that $p_k(x)$ are orthogonal, i.e. $(p_j, p_k) = 0$ for $j < k$.

   (d) Show that $\delta_i = 0$ for $i > 0$ in the three-term recursion formula for $p_i(x)$.

   (e) Show

$$\int_{-1}^{1} (x^2 - 1)^j dx = \frac{(-1)^j 2^{2j+1}}{\begin{pmatrix} 2j \\ j \end{pmatrix} (2j + 1)}$$

   Hint: use integration by parts and $(x^2 - 1) = (x + 1)(x - 1)$.

   (f) Calculate (p_j, p_j).

   (g) Show that

$$\gamma_i^2 = \frac{i^2}{(2i + 1)(2i - 1)}$$

   in the three-term recursion formula for $p_i(x)$.