MATH 5510 – Numerical Analysis 1. December 3, 2013

Final Project 2

1. Show that if $v \in \mathbb{R}^N$ and $v^T v = 1$, then the matrix $Q = I - 2vv^T$ is both symmetric and orthogonal.

2. Show that for any $x \in \mathbb{R}^N$
$$\lim_{p\to\infty} \|x\|_p = \max_{1\le i\le n} |x_i|.$$

3. Show that the following equations have a solution on the interval $[0, 1.6]$. Using the bisection method determine the roots with an error less than 10^{-4}. Repeat the problem with the Newton's Method starting from $x = 1$
$$(a) \quad 2x = e^{-x} \qquad\qquad (b) \quad 1 - x = e^{-2x}.$$

4. Consider the fix point iteration $x_{n+1} = \phi(x_n)$, where $\phi(x) = x + (x-1)^2$. Show that it has a fix point. What is it? Show that the iteration converges for $x_0 < 1$.

5. Assume that for the nonsingular matrix $A_{n-1} \in \mathbb{R}^{(N-1)\times(N-1)}$ we know the LU factorization $A_{n-1} = L_{n-1}U_{n-1}$. Compute the LU factorization for the matrix $A_n \in \mathbb{R}^{N\times N}$,
$$A_n = \begin{pmatrix} A_{n-1} & b \\ c^T & a_{nn} \end{pmatrix} = \begin{pmatrix} L_{n-1} & 0 \\ l^T & 1 \end{pmatrix} \begin{pmatrix} U_{n-1} & u \\ 0 & u_{nn} \end{pmatrix}.$$
Here, $b, c \in \mathbb{R}^{N-1}$ and $a_{nn} \in \mathbb{R}$ are given and $l, u \in \mathbb{R}^{N-1}$ and $u_{nn} \in \mathbb{R}$ are to be determined.

6. It is suspected that the data (t_i, f_i), $i = 1, \ldots, m$ obeys a relationship
$$f(t) = c_1 e^{c_2 t},$$

for some c_1 and c_2, where

t	f
0	3.8396
0.1034	4.1518
0.2069	4.1505
0.3103	4.5318
0.4138	4.4482
0.5172	4.7243
0.6207	4.8406
0.7241	4.8784
0.8276	4.9102
0.9310	5.2830
1.0345	5.3545
1.1379	5.6890
1.2414	5.8557
1.3448	6.1572
1.4483	6.2358
1.5517	6.3070
1.6552	6.6102
1.7586	6.6784
1.8621	6.9911
1.9655	7.2086
2.0690	7.4408
2.1724	7.6436
2.2759	8.0268
2.3793	7.9795
2.4828	8.4671
2.5862	8.7794
2.6897	9.0369
2.7931	9.3041
2.8966	9.5418
3.0000	9.9061

Set up a least square problem

$$\min \|Ax - b\|_2^2$$

where $A \in R^{m\times 2}$ and $b \in R^m$ and solve it using the QR-decomposition.

Plot on the same screen the least square solution function together with data points.