

Assignment 8

1. (20 points) In class to approximate derivatives of the function at a point we used the difference formulas, that we show to satisfy

$$u'(x) = \frac{u(x + \Delta x) - u(x - \Delta x)}{2\Delta x} + O(\Delta x^2)$$

and

$$u''(x) = \frac{u(x + \Delta x) - 2u(x) + u(x - \Delta x)}{\Delta x^2} + O(\Delta x^2)$$

To obtain more accurate schemes one needs to consider more terms.

Using the Taylor formula show

(a)

$$u'(x) = \frac{-u(x + 2\Delta x) + 8u(x + \Delta x) - 8u(x - \Delta x) + u(x - 2\Delta x)}{12\Delta x} + O(\Delta x^4)$$

and

(b)

$$u''(x) = \frac{-u(x + 2\Delta x) + 16u(x + \Delta x) - 30u(x) + 16u(x - \Delta x) - u(x - 2\Delta x)}{12\Delta x^2} + O(\Delta x^4).$$

2. (20 points) Write the Matlab function

```
Y = my_finite_diff(N, a, b, alpha, beta, Pfun, Qfun, Ffun)
```

that solves the two point boundary value problem

$$-u''(x) + p(x)u'(x) + q(x)u(x) = f(x), \quad a < x < b, \quad u(a) = \alpha, u(b) = \beta$$

with Finite Difference method. The variables are

- $N + 1$ - number of point
- (a, b) - any interval
- α, β - any real numbers
- Pfun, Qfun, Ffun - Matlab inline functions

Once you write the function, to check your code run the following script

```
% hw8 check
fprintf([' dx          error          rate of convergence          \n' ])
disp('-----')
a = 0;
b = 2;
alpha = 0;
beta = 4*exp(-2);
Pfun = inline('x','x');
Qfun = inline('exp(x)','x');
Ffun = inline('exp(-x).*(-2+x.^2-x.^3+4*x)+x.^2','x');
u_ex = inline('x.^2.*exp(-x)','x');
```

```

error = zeros(10,1);
N = 20;
Y = my_finite_diff(N, a, b, alpha, beta, Pfun, Qfun, Ffun);
x = linspace(a,b,N+1)';
U_ex = feval(u_ex, x);
error(1) = norm(U_ex-Y,inf);
plot(x,U_ex,x,Y,'r*')
for j = 2:8
    N = 10*2^j;
    Y = my_finite_diff(N, a, b, alpha, beta, Pfun, Qfun, Ffun);
    x = linspace(a,b,N+1)';
    U_ex = feval(u_ex, x);
    error(j) = norm(U_ex-Y,inf);
    dx = 2/(10*2^j);
    fprintf([' %1.2e  %25.3e  %25.5f  \n'],...
        dx, error(j), log2(error(j-1)/error(j)) )
end

```

Make sure that the names of your functions match. The above script can be uploaded from the course website. Print the table and the figure and submit it with the assignment.