

Assignment 6

1. (10 points) Show that if $\|\cdot\|$ is any natural norm, then

$$(\|A^{-1}\|)^{-1} \leq |\lambda| \leq \|A\|$$

for any eigenvalue λ of the nonsingular matrix A .

2. (20 points) Let the sequence $\{\mathbf{x}^{(k)}\}_{k=1}^{\infty}$ defined $\mathbf{x}^{(k)} = T\mathbf{x}^{(k-1)} + \mathbf{c}$ for $\|T\| < 1$ converges to a vector $\mathbf{x} \in \mathbb{R}^n$, with $\mathbf{x} = T\mathbf{x} + \mathbf{c}$, where $\mathbf{c}$ is a given vector and $\mathbf{x}^{(0)} \in \mathbb{R}^n$. Show the following error bounds:

$$\|\mathbf{x} - \mathbf{x}^{(k)}\| \leq \|T\|^k \|\mathbf{x}^{(0)} - \mathbf{x}\| \quad \text{and} \quad \|\mathbf{x} - \mathbf{x}^{(k)}\| \leq \frac{\|T\|^k}{1 - \|T\|} \|\mathbf{x}^{(1)} - \mathbf{x}^{(0)}\|.$$

3. (20 points) Write two Matlab functions

```
[x,k]=my_Jacobi(A,b,tolx)
```

```
[x,k]=my_Gauss_Seidel(A,b,tolx)
```

that solve the linear system $Ax = b$ approximately with Jacobi and Gauss-Seidel methods defined in Chapter 7.3 of the textbook. The input of your function should be $n \times n$ matrix A , n -vector b , and a real number $tolx$. The output should be the approximate solution x and the number of iterations k . The stoppage criteria to use is $\|Ax^{(k)} - b\|_2 \leq tolx$.

Once you write these function, check your code by running the following code

```
% hw6_check
m = [100 500 1000];
fprintf(['          Jacobi          Gauss-Seidel          \n' ])
fprintf(['n      norm(Ax-b)  iterations      norm(Ax-b)  iterations  \n' ])
disp('-----')
for j = 1:3
    n = m(j);
    A = 6*diag(ones(n,1)) - diag(ones(n-1,1),1) - ...
        diag(ones(n-1,1),-1) - diag(ones(n-2,1),2) ;
    b = (1:n)';
    % Jacobi method
    [xJ,kJ]=my_Jacobi(A,b,1e-5);
    % Gauss-Seidel method
    [xG,kG]=my_Gauss_Seidel(A,b,1e-5) ;
    fprintf([' %1.0f      %15.4e      %15.0f      %18.4e      %15.0f      \n'], ...
        n, norm(A*xJ-b), kJ, norm(A*xG-b), kG )
end
```

Make sure that the names of your functions match. The above code can be uploaded from the course website.