

### Assignment 4

1. (10 points) Find the permutation matrix  $P$  so that  $PA$  can be factored into the product  $LU$ , where  $L$  is lower triangular with 1s on its diagonal and  $U$  is upper triangular for the matrix

$$A = \begin{pmatrix} 0 & 2 & -1 \\ 1 & -1 & 2 \\ 1 & -1 & 4 \end{pmatrix}$$

2. (10 points) How the `lu` matrix factorization  $LU = PA$  can be used to compute the determinant of  $A$ ?
3. (30 points) For  $n = 100, 500, 1000$ , solve this tridiagonal system of equations

$$\begin{aligned} 2x_1 - x_2 &= 1, \\ -x_{j-1} + 2x_j - x_{j+1} &= j, \quad j = 2, \dots, n-1 \\ -x_{n-1} + 2x_n &= n, \end{aligned}$$

three different ways:

- (a) Use MATLAB's `diag` to form the coefficient matrix and then use the backslash operator to solve the system.
- (b) Use MATLAB's `spdiags` to form a sparse representation of the coefficient matrix and then use the backslash operator to solve the system.
- (c) Use `[d,e,f]=my_tridiagonal_decomposition(a,b,c)` to decompose the coefficient matrix. Then write your own MATLAB m-functions that solves the bi-diagonal systems  $Ax = b$  and  $Bx = b$ , where

$$A = \begin{pmatrix} 1 & & & & & \\ c_1 & 1 & & & & \\ & \ddots & \ddots & & & \\ & & c_{n-2} & 1 & & \\ & & & c_{n-1} & 1 & \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} d_1 & e_1 & & & & \\ & d_2 & e_2 & & & \\ & & \ddots & \ddots & & \\ & & & d_{n-1} & e_{n-1} & \\ & & & & d_n & \end{pmatrix}$$

by back and front substitutions. Your functions should take as an input vectors

$$[c_1; \dots; c_{n-1}], \quad [d_1; \dots; d_n], \quad [e_1; \dots; e_n], \quad [b_1; \dots; b_n],$$

and output vector

$$[x_1; \dots; x_n].$$

such that  $Ax = b$  or  $Bx = b$ .

Use the MATLAB's `tic-toc` commands to report the times it took to solve the problem for each  $n$ . Send me the code.