MATH 2410 – Differential Equations April 30, 2020

Practice Final Exam

No calculators. Show your work. Clearly mark each answer.

1. S Consider the following autonomous differential equation

$$y' = (y+1)(y-3)^2(y-5).$$

   (a) Compute the equilibrium solutions.

   (b) Sketch the phase line and classify the equilibria as sinks, sources, or nodes.

   (c) Describe the long term behavior of the solution to the above differential equation with initial condition $y(0) = 0$ and $y(0) = -2$.

2. Sketch the slope filed of the following differential equation

$$y' = x - y.$$

3. A five gallon tank has 1 gallon of pure water. We open a spigot so 1 gal/min. leaves the tank and at the same time introduce a mixture of 1/2 lb. per gal at 2 gal per minute. Assuming the mixture is well mixed, what is the concentration at the time when the tank is full?

4. Solve the initial value problem

$$y' - \frac{3y}{t+1} = (t+1)^2$$
$$y(0) = 3.$$

5. The following system describe a pair of competing species. Describe the long-time likely outcome of the competition by plotting the direction field.

$$\frac{dx}{dt} = x(1-x-y)$$
$$\frac{dy}{dt} = y(2-3x-y).$$

   Draw the curves $x(t)$ and $y(t)$ if $x(0) = 10$ and $y(0) = 1$ in the phase plane.

6. Find the solution to the following linear system

$$\frac{dx}{dt} = 2 - 2x$$
$$\frac{dy}{dt} = -x - 2y$$

   with initial position $x(0) = 1$ and $y(0) = 1$.

7. Consider the following second order equation

$$y'' + 6y' + 34y = 2e^{-t}.$$

(a) Compute the solution to the above equation if $y(0) = 0$, $y'(0) = 0$.

(b) Describe (in words) the long term behavior of the mass.

8. Find the general solution for the problem

$$y'' + 4y = \sin(2t).$$

Solve with initial conditions $y(0) = 0$, $y'(0) = 1$.

9. Using the Laplace transform solve the following initial value problem

$$y' + 6y = e^{-2t} + 2, \quad y(0) = 2.$$

10. Using the Laplace transform solve the following initial value problem

$$y' + 9y = 1 + H_2(t), \quad y(0) = 1,$$

where $H_2(t)$ is the Heavyside function,

$$H_2(t) = \begin{cases} 0, & 0 \leq t < 2 \\ 1, & t \geq 2. \end{cases}$$