

Practice Final Exam

No calculators. Show your work. Clearly mark each answer.

1. S Consider the following autonomous differential equation

$$y' = (y + 1)(y - 3)^2(y - 5).$$

- Compute the equilibrium solutions.
 - Sketch the phase line and classify the equilibria as sinks, sources, or nodes.
 - Describe the long term behavior of the solution to the above differential equation with initial condition $y(0) = 0$ and $y(0) = -2$.
2. Sketch the slope field of the following differential equation

$$y' = x - y.$$

3. A five gallon tank has 1 gallon of pure water. We open a spigot so 1 gal. leaves the tank and introduce a mixture of 1/2 lb. per gal at 2 gal per minute. Assuming the mixture is well mixed, what is the concentration at the time when the tank is full?
4. Solve the initial value problem

$$y' - \frac{3y}{t+1} = (t+1)^2$$
$$y(0) = 3.$$

5. The following system describe a pair of competing species. Describe the long-time likely outcome of the competition by plotting the direction field.

$$\frac{dx}{dt} = x(1 - x - y)$$
$$\frac{dy}{dt} = y(2 - 3x - y).$$

Draw the curves $x(t)$ and $y(t)$ if $x(0) = 10$ and $y(0) = 1$ in the phase plane.

6. Find the solution to the following linear system

$$\frac{dx}{dt} = 2 - 2x$$
$$\frac{dy}{dt} = -x - 2y$$

with initial position $x(0) = 1$ and $y(0) = 1$.

7. Consider the following second order equation

$$y'' + 6y' + 34y = 2e^{-t}.$$

- (a) Compute the solution to the above equation if $y(0) = 0$, $y'(0) = 0$.
- (b) Describe (in words) the long term behavior of the mass.

8. Find the general solution for the problem

$$y'' + 4y = \sin(2t).$$

Solve with initial conditions $y(0) = 0$, $y'(0) = 1$.

9. Using the Laplace transform solve the following initial value problem

$$y' + 6y = e^{-2t} + 2, \quad y(0) = 2.$$

10. Using the Laplace transform solve the following initial value problem

$$y' + 9y = 1 + H_2(t), \quad y(0) = 1,$$

where $H_2(t)$ is the Heavyside function,

$$H_2(t) = \begin{cases} 0, & 0 \leq t < 2 \\ 1, & t \geq 2. \end{cases}$$