

Practice Exam 3

No calculators. Show your work. Clearly mark each answer.

1. Consider the linear system $\vec{Y}' = A\vec{Y}$, where

$$\vec{Y} = \begin{pmatrix} x \\ y \end{pmatrix} \quad \text{and} \quad A = \begin{pmatrix} -4 & -4 \\ -6 & -2 \end{pmatrix}$$

- (a) Compute the eigenvalues of A .
 - (b) Classify the equilibrium at the origin (sink, spiral source, etc). Explain your answer.
 - (c) What is the general solution to the system? Sketch the phase plane.
2. Compute the general solution to the linear system $\vec{Y}' = A\vec{Y}$, where

$$\vec{Y} = \begin{pmatrix} x \\ y \end{pmatrix} \quad \text{and} \quad A = \begin{pmatrix} -1 & 4 \\ -4 & -1 \end{pmatrix}$$

- (a) Compute the eigenvalues of A .
 - (b) Classify the equilibrium at the origin (sink, spiral source, etc). Explain your answer.
 - (c) What is the general solution to the system? Sketch the phase plane.
3. Consider the linear system $\vec{Y}' = A\vec{Y}$, where

$$\vec{Y} = \begin{pmatrix} x \\ y \end{pmatrix} \quad \text{and} \quad A = \begin{pmatrix} 1 & -3 \\ 3 & -5 \end{pmatrix}$$

- (a) Compute the eigenvalues of A .
- (b) Compute the solution to initial value problem with

$$\vec{Y}(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

4. Consider the spring-mass system whose motion is governed by

$$y'' + 6y' + 34y = 2e^{-t}.$$

- (a) Compute the solution to the above equation if $y(0) = 0$, $y'(0) = 0$.
 - (b) Describe the long term behavior of the mass.
5. Find the general solution for the damped spring-mass problem

$$y'' + 4y = \sin(2t).$$

Solve with initial conditions $y(0) = 0$, $y'(0) = 1$.