MATH 2410 – Differential Equations April 2, 2019

# Practice Exam 2

*No calculators. Show your work. Clearly mark each answer.*

1. (20 points) Find the general solution for the problem

$$\frac{dx}{dt} = x$$
$$\frac{dy}{dt} = x + 2y.$$

Solve with initial conditions $x(0) = 1$, $y(0) = 3$.

2. (20 points) The following system describe a pair of competing species. Describe the long-time likely outcome of the competition by plotting the direction field.

$$\frac{dx}{dt} = x(2 - x - y)$$
$$\frac{dy}{dt} = y(3 - x - y).$$

Draw the curves $x(t)$ and $y(t)$ if $x(0) = 0.5$, $y(0) = 1$ and $x(0) = 1$, $y(0) = 3$ in the phase plane.

3. (20 points) Consider the linear system $\vec{Y}' = A\vec{Y}$ where $\vec{Y} = (x(t), y(t))^T$

$$A = \begin{pmatrix} 4 & -2 \\ 1 & 7 \end{pmatrix}$$

Find the general solution. Sketch the solution curves in the phase plane.

4. (20 points) Consider the linear system $\vec{Y}' = A\vec{Y}$ where $\vec{Y} = (x(t), y(t))^T$

$$A = \begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix}$$

Find the general solution. Solve for $x(0) = 1$, $y(0) = 2$.

5. (20 points) A 200-gallon tank initially contains 2 pounds of sugar. Suppose water containing 0.5 sugar per gallon flows through one pipe into the tank at a rate of 5 gallons per minute. The water in the tank is kept well mixed and well-mixed solution leaves the bottom of the tank at rate 10 gallons per minute into a second 300-gallon tank that initially has no sugar. The water in the second tank is kept well mixed and well-mixed solution leaves the bottom of the tank at rate 10 gallons per minute. Make a sketch of the problem and set up the initial value problem for the amount of sugar in the both tanks at time $t$ (do not solve it).