

**Practice Exam 2**

*No calculators. Show your work. Clearly mark each answer.*

1. (20 points) Consider the equation

$$y'(t) = (2 - y)(1 + y).$$

- (a) Solve with initial conditions  $y(0) = 1$ .
- (b) What is the long time behaviour of the solution with  $y(0) = 1$ , i.e. compute  $\lim_{t \rightarrow \infty} y(t)$ .
- (c) Confirm your answer by sketching the slope field of the equation.

2. (20 points) Find the general solution for the problem

$$\begin{aligned}\frac{dx}{dt} &= x \\ \frac{dy}{dt} &= x + 2y.\end{aligned}$$

Solve with initial conditions  $x(0) = 0$ ,  $y(0) = 1$ .

3. (20 points) The following system describe a pair of competing species. Describe the long-time likely outcome of the competition by plotting the direction field.

$$\begin{aligned}\frac{dx}{dt} &= x(2 - x - y) \\ \frac{dy}{dt} &= y(3 - x - y).\end{aligned}$$

Draw the curves  $x(t)$  and  $y(t)$  if  $x(0) = 1$ ,  $y(0) = 1$  and  $x(0) = 10$ ,  $y(0) = 1$  in the phase plane.

4. (20 points)

A person opens a savings account with an initial deposit of \$10,000 and subsequently deposits \$100 each month. Find the value of the account at time  $t > 0$ , assuming that the bank pays 1% interest compounded continuously.

5. (20 points) Compute the Euler's approximate solution at time  $t = 1$  of the following equation

$$y'(t) = y(2 - y).$$

With initial position  $y(0) = 1$  and time step  $\Delta t = 0.5$