

Practice Exam 2

No calculators. Show your work. Clearly mark each answer.

1. (20 points) Find the general solution for the system

$$y'' - 5y' + 4y = 1 + t$$

Solve with initial conditions $y(0) = 1$, $y'(0) = 0$.

2. (20 points) Find the general solution for the problem

$$\begin{aligned}\frac{dx}{dt} &= x + 2y \\ \frac{dy}{dt} &= -y.\end{aligned}$$

Solve with initial conditions $x(0) = 1$, $y(0) = 2$.

3. (20 points) The following system describe a pair of competing species. Describe the long-time likely outcome of the competition by plotting the direction field.

$$\begin{aligned}\frac{dx}{dt} &= x(1 - x - y) \\ \frac{dy}{dt} &= y(2 - 4x - y).\end{aligned}$$

Draw the curves $x(t)$ and $y(t)$ if $x(0) = 0.5$, $y(0) = 2$ and $x(0) = 2$, $y(0) = 0.5$ in the phase plane.

4. (20 points) Consider the linear system $\vec{Y}' = A\vec{Y}$ where $\vec{Y} = (x(t), y(t))^T$

$$A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$$

Find the general solution. Solve for $x(0) = 1$, $y(0) = 2$.

5. (20 points) Compute the Euler's approximate solution at time $t = 1$ of the following system

$$\begin{aligned}\frac{dx}{dt} &= xy + 2t \\ \frac{dy}{dt} &= y - x.\end{aligned}$$

With initial position $x(0) = 1$ and $y(0) = 0$ and time step $\Delta t = 0.5$