MATH 2410 – Differential Equations February 18, 2020

Practice Exam 1

No calculators. Show your work. Clearly mark each answer.

1. (15 points) Consider the autonomous differential equation

$$\frac{dy}{dt} = y(y-3)^2(y-5).$$

   (a) Compute the equilibrium solutions.

   (b) Sketch the phase line and classify the equilibria as sinks, sources, or nodes.

   (c) Describe the long term behavior of the solution to the above differential equation with initial condition $y(0) = 4$ and $y(0) = 1$.

2. (15 points) Consider the linear equation

$$2y' + y = e^x$$

   (a) Find the 1-parameter family of solution of the differential equation.

   (b) Find the solution of the differential equation with the given initial value $y(0) = \alpha$.

   (c) For what value(s) of α, the solution you found in (b) remains finite as $x \to \infty$?

3. (20 points) Consider the equation.

$$\frac{dy}{dt} = 2y(t^2+1).$$

   (a) Find the general solution of the above equation

   (b) Using the Euler method approximate $y(1)$ with initial condition $y(0) = 1$ and the time step $\Delta t = 0.5$.

4. (15 points) Consider the following differential equation

$$(x^2y + y)\frac{dy}{dx} = -(xy^2 + x^2)$$

   (a) Show that the above equation is exact

   (b) Find the 1-parameter family of solution of the differential equation in implicit form.

   (c) Find the particular solution to the initial value problem $y(0) = 2$.

5. (15 points) Solve the following Bernoulli equation

$$xy' + y + x^2y^2e^x = 0.$$

   by using the substitution $u = y^{-1}$.

6. (20 points) A 400-gallon tank initially contains 1 pound of sugar. Suppose water containing 0.5 pounds of sugar per gallon flows into the top of the tank at a rate of 2 gallons per minute. The water in the tank is kept well mixed and well-mixed solution leaves the bottom of the tank at the same rate, 2 gallons per minute. How much sugar will be in the tank after 10 minutes? What does the concentration approach in the long run?