MATH 2410 – Differential Equations February 6, 2018

Practice Exam 1

No calculators. Show your work. Clearly mark each answer.

1. (20 points) Consider the autonomous differential equation

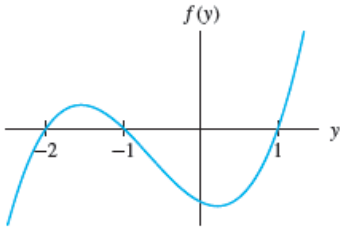
$$\frac{dy}{dt} = y(y-3)^2(y-5).$$

   (a) Compute the equilibrium solutions.

   (b) Sketch the phase line and classify the equilibria as sinks, sources, or nodes.

   (c) Describe the long term behavior of the solution to the above differential equation with initial condition $y(0) = 4$ and $y(0) = 1$.

2. (20 points)



Consider the autonomous differential equation

$$\frac{dy}{dt} = f(y),$$

with the graph of $f(y)$ on the right. Sketch the phase line and classify the equilibria as sinks, sources, or nodes.

3. (20 points) Consider the equation.

$$\frac{dy}{dt} = 2y(t^2+1).$$

   (a) Find the general solution of the above equation

   (b) Using the Euler method approximate $y(1)$ with initial condition $y(0) = 1$ and the time step $\Delta t = 0.5$.

4. (20 points) Consider the differential equation

$$\frac{dy}{dt} = yt^{\frac{2}{5}}.$$

   (a) Compute the general solution to the above differential equation.

   (b) Is there a unique solution $y(t)$ to the above differential equation such that $y(0) = 0$? Why or why not?

(c) Is there a unique solution y(t) to the above differential equation such that $y(0) = 1$? Why or why not?

5. (20 points) A 400-gallon tank initially contains 1 pound of sugar. Suppose water containing 0.5 pounds of sugar per gallon flows into the top of the tank at a rate of 2 gallons per minute. The water in the tank is kept well mixed and well-mixed solution leaves the bottom of the tank at the same rate, 2 gallons per minute. How much sugar will be in the tank after 10 minutes? What does the concentration approach in the long run?