

Practice Exam 1

No calculators. Show your work. Clearly mark each answer.

1. (20 points) Check if the given function y is a solution of the initial value problem

(a) $y = x^2(1 + \ln x)$

$$y''(x) = \frac{3xy'(x) - 4y}{x^2}, \quad y(e) = 2e^2, \quad y'(e) = 5e.$$

(b) $y = \frac{x^2}{3} + x - 1$

$$y''(x) = \frac{x^2 - xy'(x) + y + 1}{x^2}, \quad y(1) = \frac{1}{3}, \quad y'(1) = \frac{5}{3}.$$

2. (20 points) Find the general solution of

$$\frac{dy}{dt} + 2y = t^2 + 1.$$

3. (20 points) Solve the initial value problem

$$\begin{aligned} \frac{dy}{dt} + \frac{2}{t}y &= t^2 \\ y(1) &= 0. \end{aligned}$$

4. (20 points) Consider the differential equation

$$\frac{dy}{dt} = yt^{\frac{2}{5}}.$$

- (a) Compute the solution to the above differential equation.
- (b) Is there a unique solution $y(t)$ to the above differential equation such that $y(0) = 0$? Why or why not?
- (c) Is there a unique solution $y(t)$ to the above differential equation such that $y(0) = 1$? Why or why not?
5. (20 points) A 400-gallon tank initially contains 200 gallons of sugar water at concentration of 0.1 pounds of sugar per gallon. Suppose water containing 0.5 sugar per gallon flows into the top of the tank at a rate of 2 gallons per minute. The water in the tank is kept well mixed and well-mixed solution leaves the bottom of the tank at rate 1 gallon per minute. How much sugar is in the tank when the tank is full?