

**Practice Final Exam**

*No calculators. Show your work. Clearly mark each answer.*

1. Sketch the slope field of the autonomous differential equation

$$y' = (y + 1)(y - 3)^2(y - 5).$$

2. Sketch the slope field of the following differential equation

$$y' = x - y.$$

3. A five gallon tank has 1 gallon of pure water. We open a spigot so 1 gal. leaves the tank and introduce a mixture of 1/2 lb. per gal at 2 gal per minute. Assuming the mixture is well mixed, what is the concentration at the time when the tank is full?

4. Solve the initial value problem

$$\begin{aligned}y' - \frac{3y}{t+1} &= (t+1)^2 \\ y(0) &= 3.\end{aligned}$$

5. The following system describe a pair of competing species. Describe the long-time likely outcome of the competition by plotting the direction field.

$$\begin{aligned}\frac{dx}{dt} &= x(1 - x - y) \\ \frac{dy}{dt} &= y(2 - 3x - y).\end{aligned}$$

Draw the curves  $x(t)$  and  $y(t)$  if  $x(0) = 10$  and  $y(0) = 1$  in the phase plane.

6. Compute the Euler's approximate solution at time  $t = 1$  of the following system

$$\begin{aligned}\frac{dx}{dt} &= x(1 - x - y) \\ \frac{dy}{dt} &= y(1 - x - 2y).\end{aligned}$$

With initial position  $x(0) = 2$  and  $y(0) = 1$  and time step  $\Delta t = 0.5$ .

7. Find the solution to the following linear system

$$\begin{aligned}\frac{dx}{dt} &= 2 - 2x \\ \frac{dy}{dt} &= -x - 2y\end{aligned}$$

with initial position  $x(0) = 1$  and  $y(0) = 1$ .

8. Consider the following second order equation

$$y'' + 6y' + 34y = 2e^{-t}.$$

- (a) Compute the solution to the above equation if  $y(0) = 0$ ,  $y'(0) = 0$ .
- (b) Describe (in words) the long term behavior of the mass.

9. Find the general solution for the damped spring-mass problem

$$y'' + 4y = \sin(2t).$$

Solve with initial conditions  $y(0) = 0$ ,  $y'(0) = 1$ .

10. Using the Laplace transform solve the following initial value problem

$$y' + 6y = e^{-2t} + 2, \quad y(0) = 2.$$

11. Using the Laplace transform solve the following initial value problem

$$y' + 9y = 1 + H_2(t), \quad y(0) = 1,$$

where  $H_2(t)$  is the Heavyside function,

$$H_2(t) = \begin{cases} 0, & 0 \leq t < 2 \\ 1, & t \geq 2. \end{cases}$$