

**Assignment 5**

1. Find the Laplace transform of the following function

$$f(x) = e^x + 2e^{2x}.$$

2. Find the Laplace transform of the following function

$$f(x) = \begin{cases} 1, & 0 \leq x < 1 \\ e^x, & x \geq 1. \end{cases}$$

3. Show that the Laplace transform of  $f(x) = e^{x^2}$  does not exist, i.e. that

$$\int_0^{\infty} e^{x^2} e^{-sx} dx = \infty, \quad \text{for any } s > 0.$$

4. Using the Laplace transform solve the following initial value problem

$$y'(x) = 5 - 3y(x), \quad y(0) = 2.$$

5. Using the Laplace transform solve the following initial value problem

$$y'(x) = e^x - 3y(x), \quad y(0) = 3.$$

6. Using the Laplace transform solve the following initial value problem

$$y'(x) = e^{-2x} + 2y(x), \quad y(0) = 1.$$