MATH 2210 – Applied Linear Algebra December 9, 2015

Practice Final Exam

1. Find the standard matrix for the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^2$ such that

$$T\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad T\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad T\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}.$$

2. True or False. If $T : \mathbb{R}^2 \to \mathbb{R}^2$ rotates vectors about the origin though an angle $\pi/10$, then T is a linear transformation. Explain.

3. Find the eigenvalues and the eigenvectors of the matrix

$$\begin{pmatrix} 5 & -2 & 3 \\ 0 & 1 & 0 \\ 6 & 7 & -2 \end{pmatrix}$$

4. Let

$$A = \begin{pmatrix} -2 & 12 \\ -1 & 5 \end{pmatrix}$$

Find a diagonal matrix D and an invertible matrix P such that $A = PDP^{-1}$. Compute A^{10}.

5. For the following matrices A find the basis for $Nul(A)$, $Row(A)$, $Col(A)$. What is $rank(A)$?

$$A = \begin{pmatrix} 1 & 2 & 2 & 0 & 1 \\ 0 & 2 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

6. If the null space of a 50×60 matrix A is 40-dimensional,
   (a) What is the rank of A?
   (b) $Null(A)$ is a subspace of $\mathbb{R}^n$, what is n?
   (c) $Col(A)$ is a subspace of $\mathbb{R}^n$, what is n?

7. Let A be a n-by-n matrix that satisfies $A^2 = A$. What can you say about the determinant of A?

8. Suppose a 4×7 matrix A has four pivot columns. Is Col $A = \mathbb{R}^4$? Is Nul $A = \mathbb{R}^3$? Explain.

9. Show that the set $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ is an orthogonal set in $\mathbb{R}^3$. Then express a vector $\mathbf{x}$ as a linear combination of $\mathbf{u}'s$, where

$$\mathbf{u}_1 = \begin{pmatrix} 3 \\ -3 \\ 0 \end{pmatrix}, \quad \mathbf{u}_2 = \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}, \quad \mathbf{u}_3 = \begin{pmatrix} 1 \\ 1 \\ 4 \end{pmatrix}, \quad \mathbf{x} = \begin{pmatrix} 5 \\ -3 \\ 1 \end{pmatrix}.$$

10. Using the Gram-Schmidt process to produce an orthogonal basis for $W = span\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$, where

$$\mathbf{v}_1 = \begin{pmatrix} 1 \\ -4 \\ 0 \\ 1 \end{pmatrix}, \quad \mathbf{v}_2 = \begin{pmatrix} 7 \\ -7 \\ -4 \\ 1 \end{pmatrix}, \quad \mathbf{v}_3 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix}.$$