MATH 2210 – Applied Linear Algebra November 16, 2015

Practice Exam 2

1. Find A^{-1}, where
$$A = \begin{pmatrix} 3 & 2 \\ 4 & 3 \end{pmatrix}.$$
Using A^{-1} solve the system
$$\begin{aligned} 3x_1 + 2x_2 &= 2 \\ 4x_1 + 3x_2 &= 3. \end{aligned}$$

2. Find A^{-1}, where
$$A = \begin{pmatrix} 1 & 3 & 0 \\ 0 & 1 & 3 \\ 1 & 0 & 1 \end{pmatrix}.$$
Using A^{-1} solve the system
$$\begin{aligned} x_1 + 3x_2 &= 1 \\ x_2 + 3x_3 &= 2 \\ x_1 + x_3 &= 3. \end{aligned}$$

3. Show that the following set S of polynomials is linearly independent and hence form a basis for $\mathbb{P}_2$
$$S = \{p_1(x), p_2(x), p_3(x)\},$$
where $p_1(x) = 1 - x$, $p_2(x) = 1 + x^2$, $p_3(x) = x + 2x^2$. Find the coordinates of the polynomial $p(x) = x^2 + x + 1$ in the basis S.

4. Show that T is a linear transformation by finding a matrix that implements the mapping. $T(x_1; x_2; x_3) = (2x_1 + 3x_2; 3x_1 + 2x_3; x_1 + x_2; x_2 + x_3)$.

5. For the following matrix A find the basis for $Nul(A)$, $Row(A)$, $Col(A)$, and compute the $rank(A)$.
$$A = \begin{pmatrix} 1 & 2 & 3 & 0 \\ 1 & 2 & 1 & 1 \\ 2 & 1 & 2 & 2 \end{pmatrix}.$$

6. Can a matrix A with a zero row have $Nul(A) = \{\mathbf{0}\}$? Explain.

7. If A is 100×99 matrix, what is the smallest possible dimension of the $Nul(A)$?

8. Is the following set a subspace of $\mathbb{R}^3$
$$\left\{ \begin{pmatrix} r \\ s \\ t \end{pmatrix} : 2r = s + t \quad \text{and} \quad r = t - s \right\}?$$
Explain.