MATH 1132 – Calculus II April 30, 2020

Practice Final Exam

No calculators. Show your work. Clearly mark each answer.

1. (15 points) Find the limits of the following sequences $\{a_n\}$.

   (a)
   $$a_n = \frac{n^2 - 2n + 1}{3n^3 + 1}$$

   (b)
   $$a_n = \frac{\sin n}{n^2}$$

   (c)
   $$a_n = \left(1 - \frac{1}{n}\right)^n$$

2. (20 points) Which of the following series converge or diverge? Give reasons to your answers.

   (a)
   $$\sum_{n=2}^{\infty} \frac{n^2 + 1}{(n^2 - 1)^3}$$

   (b)
   $$\sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{\sqrt{\ln n}}$$

   (c)
   $$\sum_{n=0}^{\infty} \frac{n}{n+1}$$

   (d)
   $$\sum_{n=0}^{\infty} \frac{1}{e^n}$$

3. (20 points) Find the radii and the intervals of convergence of the following power series.

   (a)
   $$\sum_{n=1}^{\infty} \frac{4^n x^n}{n^3}$$

   (b)
   $$\sum_{n=1}^{\infty} \frac{(x-3)^n}{2^n}$$

   (c)
   $$\sum_{n=0}^{\infty} \frac{x^{2n}}{5^n n}$$

(d)
$$\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n}}{n!}$$

4. (15 points) Find the Maclaurin series of the following function
$$(x^2-1)e^{2x}.$$

5. (15 points) Find the quadratic ($n=2$) Taylor polynomial at $a=1$ of the following function
$$\sqrt[4]{x}.$$

6. (10 points) What is the largest d can be such that the approximation
$$\sin x \approx x - x^3/6$$
is accurate to 5 decimal places for $|x| \le d$?

7. (15 points) Plot the points whose polar coordinates are given below
$$(1, 3\pi/2), \quad (-2, 5\pi/4), \quad (-1, -3\pi/2).$$

8. (10 points) Convert the point $(\frac{1}{2}, \frac{\sqrt{3}}{2})$ from Cartesian to polar coordinates. Give at least two different representations in polar coordinates.

9. (20 points) Let the curve C be defined by the parametric equations $x = t^2 - t$ and $y = t^3 + 1$. Find the equation of the tangent line when $t = 2$. Find the points on the curve where the tangent line is horizontal or vertical.

10. (10 points) Using Taylor formula, compute the following limits.

(a)
$$\lim_{x\to 0} \frac{e^{2x} - 1 - 2x - 2x^2}{x^3}$$

(b)
$$\lim_{x\to 0} \frac{\cos x - 1 + \frac{x^2}{2} - \frac{x^4}{24}}{x^6}$$