Practice Final Exam

No calculators. Show your work. Clearly mark each answer.

1. Find the limits of the following sequences $\{a_n\}$.

   (a)
   $$a_n = ne^{-n}$$

   (b)
   $$a_n = \frac{\cos n}{\sqrt{n}}$$

   (c)
   $$a_n = \left(1 + \frac{1}{n}\right)^{2n}$$

2. Which of the following series converge or diverge? Give reasons to your answers.

   (a)
   $$\sum_{n=2}^{\infty} \frac{n^2 + 1}{(n^2 - 1)^2}$$

   (b)
   $$\sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{n \ln n}$$

   (c)
   $$\sum_{n=0}^{\infty} \frac{n}{(n+1)^4}$$

3. Find the radii and the intervals of convergence of the following power series.

   (a)
   $$\sum_{n=1}^{\infty} \frac{3^n x^n}{n^2}$$

   (b)
   $$\sum_{n=1}^{\infty} \frac{(x - 1)^n}{2^n n}$$

   (c)
   $$\sum_{n=0}^{\infty} \frac{x^{2n}}{4^n}$$

4. Compute the following limits.

(a)
$$\lim_{x \to 0} \frac{e^{2x} - 1 - 2x}{x^2}$$

(b)
$$\lim_{x \to 0} \frac{\sin x - x + \frac{x^3}{6}}{x^5}$$

5. Find the Maclaurin series of the following function
$$(x - 1)e^{3x}.$$

6. Find the quadratic $T_2(x)$ Taylor polynomial at $a = 1$ of the following function
$$x^{5/2}.$$

7. What is the largest d can be such that the approximation
$$\cos x \approx 1 - x^2/2$$
is accurate to 4 decimal places for $|x| \le d$?

8. Plot the points whose polar coordinates are given below
$$(2, \pi/2), \quad (-1, 3\pi/4), \quad (-1, -\pi/2).$$

9. Convert the point $(1, 1)$ from Cartesian to polar coordinates. Give at least two different representations in polar coordinates.

10. Let the curve C be defined by the parametric equations $x = t^3 - t$ and $y = t^2 - 1$. Find the equation of the tangent line when $t = 2$. Find the points on the curve where the tangent line is horizontal or vertical.

11. Find the area of the region enclosed by one loop of the curve $r = \sin(4\theta)$.