

Practice Exam 2

No calculators. Show your work. Clearly mark each answer.

1. (20 points) Find the limits of the following sequences $\{a_n\}$.

(a)

$$a_n = \frac{n^2 + 5n - 1}{3n^3 + 5}$$

(b)

$$a_n = \frac{\sin n}{\sqrt[3]{n}}$$

(c)

$$a_n = \left(1 + \frac{3}{n}\right)^n$$

(d)

$$a_n = \frac{\ln n}{n}$$

2. (20 points) Which of the following series converge or diverge? Give reasons to your answers.

(a)

$$\sum_{n=2}^{\infty} \frac{n^3 + 1}{(n^3 - 1)^2}$$

(b)

$$\sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{\ln n}$$

(c)

$$\sum_{n=0}^{\infty} (-1)^n \frac{n^2}{n+1}$$

(d)

$$\sum_{n=1}^{\infty} \frac{5^n}{n!}$$

3. (20 points) Using partial fractions show that the following improper integral converges.

$$\int_1^{\infty} \frac{dx}{x^2(x+1)}$$

What can you say about convergence of the following series?

$$\sum_{n=1}^{\infty} \frac{1}{n^2(n+1)}$$

4. (20 points) Express the following number as a ratio of two integers.

$$0.113113113113\dots$$

5. (20 points) Find the value of the following series

$$\sum_{n=2}^{\infty} \left(\frac{2}{2^n} - \frac{3}{3^n} \right).$$