MATH 1131 – Calculus I April 28, 2013

Practice Final Exam

No calculators. Show your work. Clearly mark each answer.

1. State the domain, range and possible symmetries of the following functions:

   (a)
   $$\sqrt{x^2+1}$$

   (b)
   $$\sqrt{x+1}$$

   (c)
   $$\frac{x+1}{x-1}$$

2. Find the vertical and horizontal asymptotes of the following functions:

   (a)
   $$\frac{\sin x}{\sqrt{x}}$$

   (b)
   $$\frac{x+1}{x-1}$$

3. Find the equation of the tangent line at point $(2,0)$ for $e^y + x^2 = 5$.

4. Find the linear approximation of $\sin(x)$ at point $\pi/4$.

5. Evaluate the following limits:

   (a)
   $$\lim_{x\to 0} \frac{3x}{\sin(4x)}$$

   (b)
   $$\lim_{x\to\infty} \frac{x^2+2}{3x^2-4x+5}$$

   (c)
   $$\lim_{x\to 0^+} x^x$$

6. Find the absolute maximum and absolute minimum of the function $f(x) = x^3 - x + 1$ on the interval $[0,1]$.

7. The Riemann sum for a function $f(x)$ on the interval $[a,b]$ for an arbitrary n is
   $$\sum_{k=1}^{n} f(\bar{x}_k)\Delta x,$$
   where $\Delta x = \frac{b-a}{n}$.
   For the left Riemann sum
   $$\bar{x}_k = a + (k-1)\Delta x$$

and for the right Riemann sum

$$\bar{x}_k = a + k\Delta x.$$

Write the left and right Riemann sums for the function $f(x) = \frac{4}{x}$ on the interval $[2, 4]$ for $n = 4$. What can you say about $\int_2^4 f(x)dx$?

8. Using the Fundamental Theorem of Calculus find the following derivatives:

   (a)
   $$\frac{d}{dx}\int_1^x t^2 dt$$

   (b)
   $$\frac{d}{dx}\int_{x^2}^2 t^2 dt$$

9. Find the following antiderivatives:

   (a)
   $$\int (x^2 - 1)dx$$

   (b)
   $$\int \frac{dx}{2x^2 + 1}dx$$

   hint: $\int \frac{dx}{x^2+1}dx = \tan^{-1} x + C$

   (c)
   $$\int (x^2 + 2)^2 dx$$

10. Using the Fundamental Theorem of Calculus compute the following integrals:

    (a)
    $$\int_1^2 x^3 dx$$

    (b)
    $$\int_0^{e-1} \frac{dx}{x + 1}dx$$