MATH 1050QC
Mathematical Modeling in the Environment
Lecture 6. Interstitial Velocity Equation.

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General Issues

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1. the quantity of water flowing through the aquifer
2. the speed or velocity of the flow

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Darcy's Law give us the answer to the first question and we turn to the second question.

Flow through open channel with unit cross-sectional area

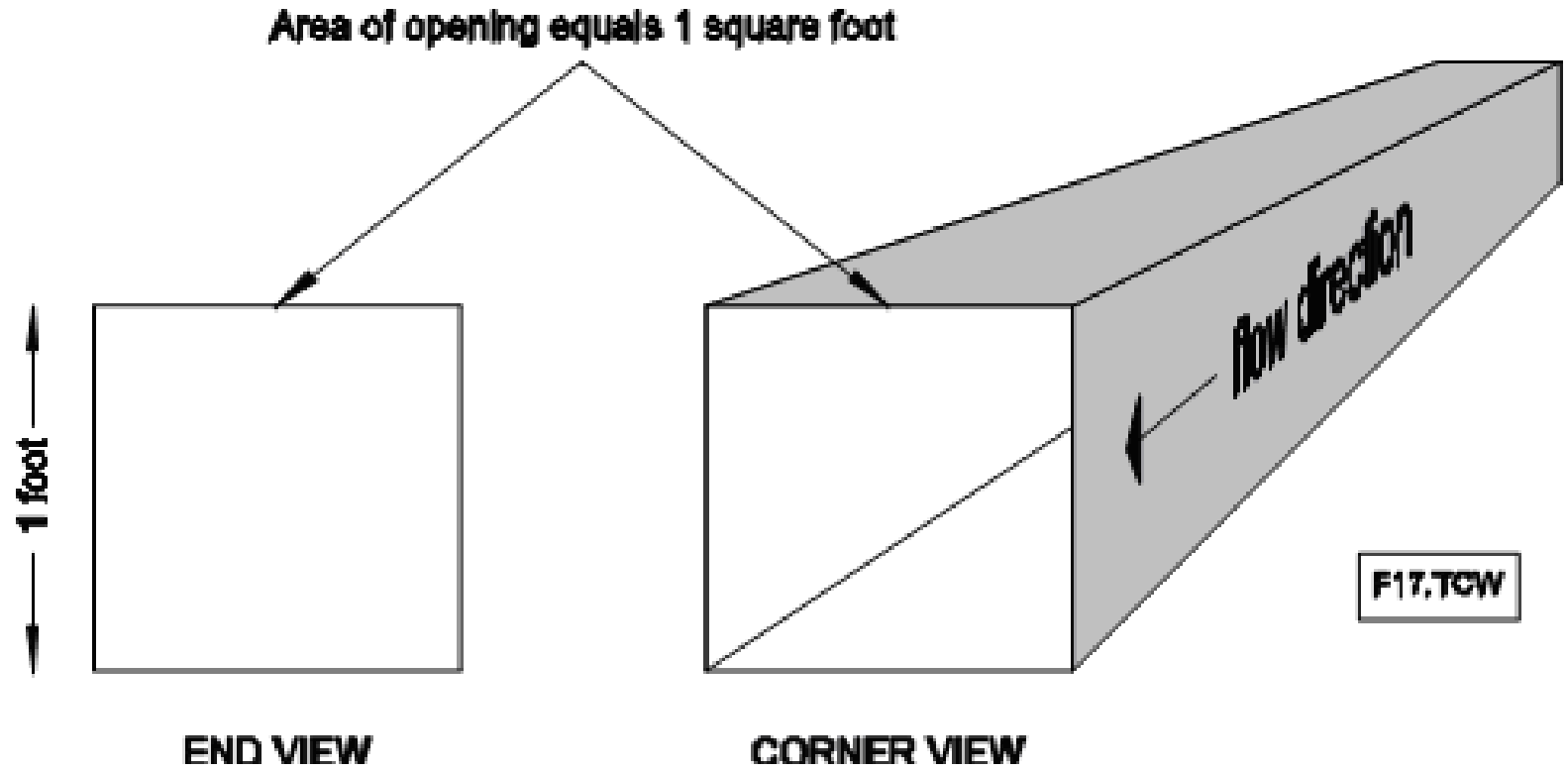

figure 2.17 from C. Hadlock's book

Flow through open channel with half cross-sectional area blocked

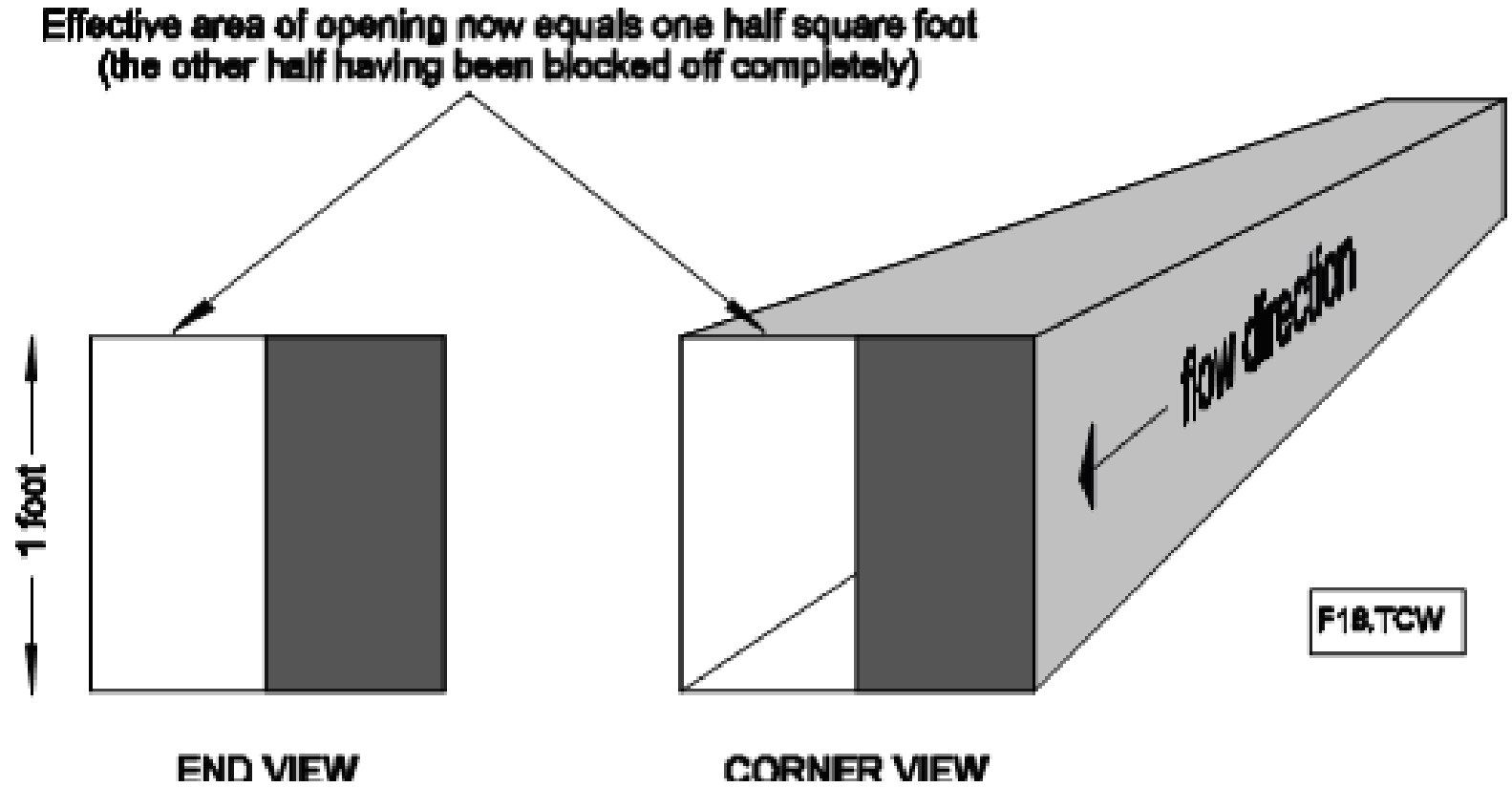

figure 2.18 from C. Hadlock's book

Flow through channel filled with porous material with porosity $\eta = 0.5$

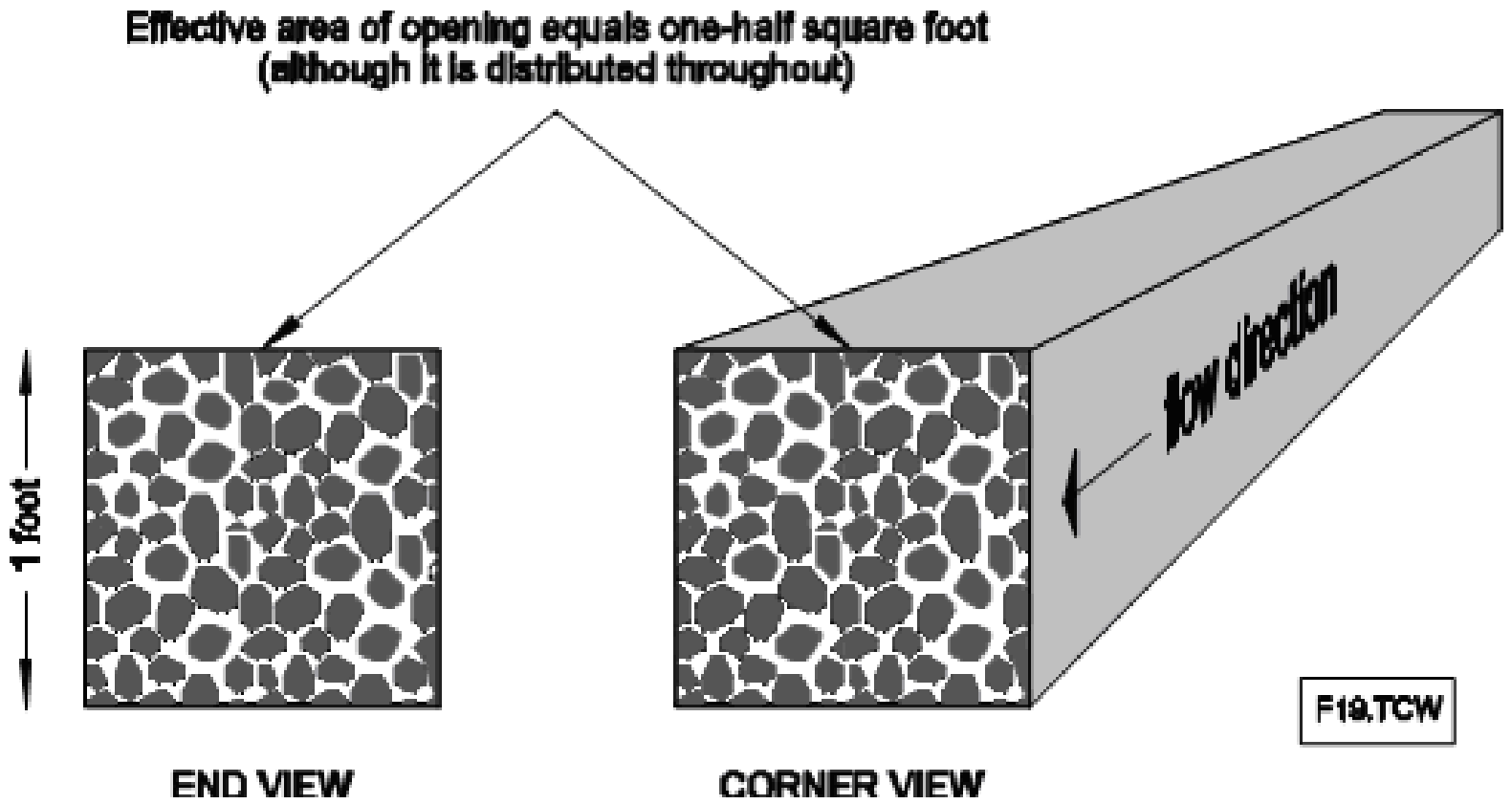

figure 2.19 from C. Hadlock's book

Interstitial Velocity

Definition

Interstitial Velocity is the speed at which the molecules of water are progressing in the direction of movement.

Let v represent interstitial velocity

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Let η denote porosity, then

$$v = \frac{q}{\eta} = \frac{Ki}{\eta}$$

- v-the effective fluid velocity along the flow path
- K-hydraulic conductivity
- i - hydraulic gradient
- η the porosity of medium (see table)

Typical ranges of values of porosity and hydraulic conductivity

Geological Medium	**Porosity**, η	**Hydraulic Conductivity** K (ft/day)
Gravel	0.25-0.40	100-100,100
Sand	0.25-0.50	0.01-1000
Silt	0.35-0.50	0.001-0.1
Clay	0.40-0.70	0.0000001-0.001
Sandstone	0.05-0.30	0.00001-0.1
Limestone	0.001-0.20	0.0001-0.1
Granite (fractured)	0.0001-0.10	0.0001-10

Example. Block diagram of idealized aquifer

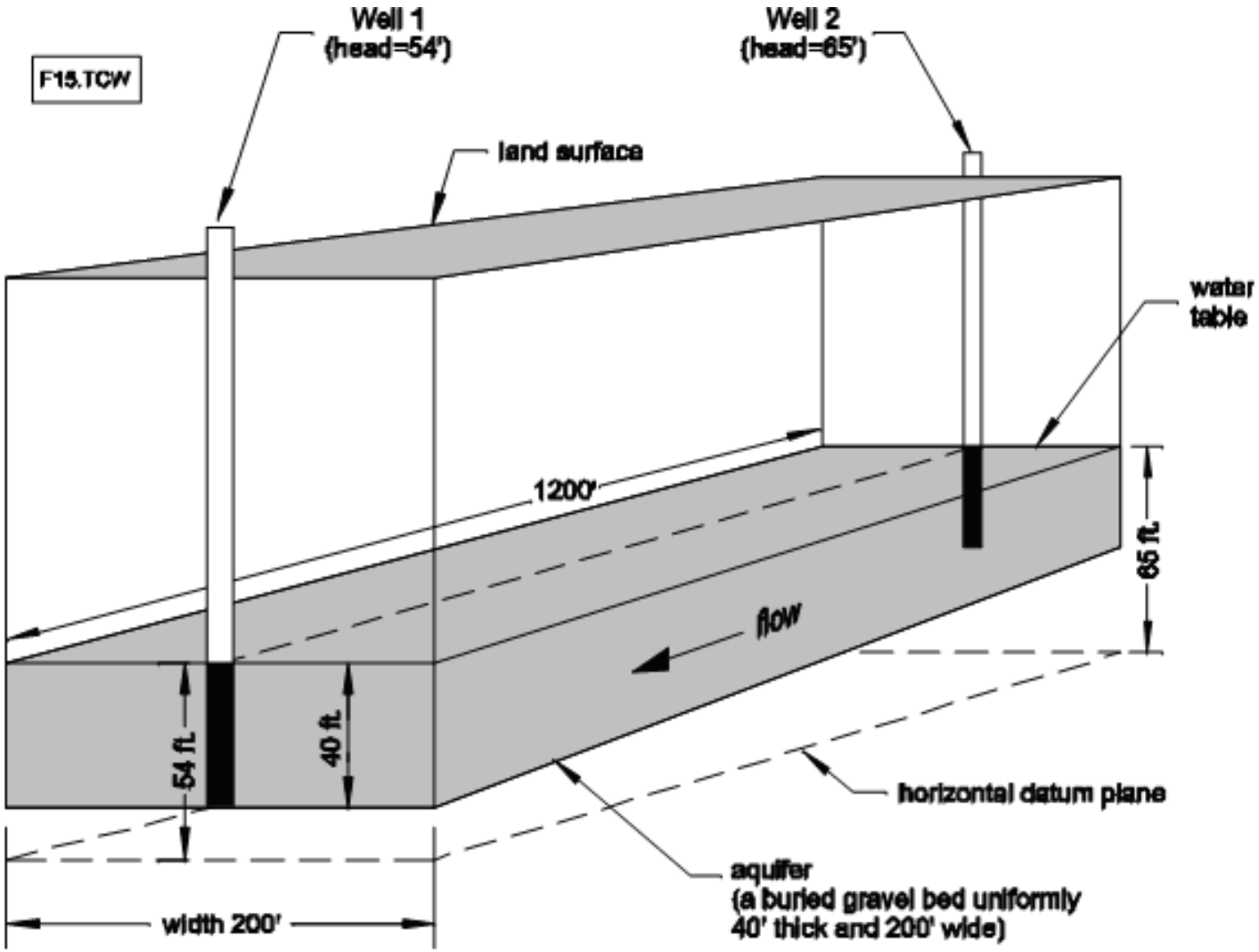

figure 2.15 from C. Hadlock's book

Example

Assume the porosity of the geological sample is $\eta = 0.3$ and the hydraulic conductivity is $K = 100$ ft/day. Then

$$\begin{aligned} v &= \frac{Ki}{\eta} \\ &= 100 \text{ ft/day} \times \frac{65 \text{ ft} - 54 \text{ ft}}{1200 \text{ ft}} \times \frac{1}{0.3} \\ &\approx 3.06 \text{ ft/day}, \end{aligned}$$

which is rather slow.

Example (cont.)

Thus since

$$\text{time} = \frac{\text{distance}}{\text{velocity}}$$

and

$$1 \text{ mile} = 5,280 \text{ ft},$$

it will take

$$\frac{5,280 \text{ ft}}{3.06 \text{ ft/day}} = 1,725 \text{ days}$$

to travel one mile.