

Assignment 5

1. (10 points)

Use **fzero** to try to find a zero of each of the following functions in the given interval. Do you see any interesting or unusual behavior?

(a) $\text{atan}(x) - \pi/3$ on $[0, 5]$

(b) $1/(x - \pi)$ on $[0, 5]$

(c) $\text{sign}(x - 2)\sqrt{|x - 2|}$ on $[1, 4]$

2. (30 points) Here is a cubic polynomial with three closely spaced real roots.

$$p(x) = 816x^3 - 3835x^2 + 6000x - 3125$$

(a) What are the exact roots of p ?

(b) Plot $p(x)$ for $1.43 < x < 1.71$. Show the location of the three roots.

(c) Starting with $x_0 = 1.5$, what does Newton's method do?

(d) Starting with the interval $[1, 2]$, what does bisection do?

(e) What is **fzero**($p, [1, 2]$)? Why?

3. (20 points) Utilities must avoid freezing water mains. If we assume uniform soil conditions, the temperature $T(x, t)$ at a distance x below the surface and time t after the beginning of a cold snap is given approximately by

$$\frac{T(x, t) - T_s}{T_i - T_s} = \text{erf}\left(\frac{x}{2\sqrt{\alpha t}}\right)$$

Here T_s is the constant surface temperature during the cold period, T_i is the initial soil temperature before the cold snap, and α is the thermal conductivity of the soil. If x is measured in meters and t in seconds, then $\alpha = 0.138 * 10^{-6} m^2/s$. Let $T_i = 20^\circ C$, $T_s = -15^\circ C$, and recall that water freezes at $0^\circ C$. Use **fzero** to determine how deep a water main should be buried so that it will not freeze until at least 60 days exposure under these conditions.