

Assignment 4

1. (20 points)

It is suspected that the data (t_i, f_i) , $i = 1, \dots, m$ obeys a relationship

$$f(t) = c_1 e^{c_2 t},$$

for some c_1 and c_2 , where

t	f
0	3.8396
0.1034	4.1518
0.2069	4.1505
0.3103	4.5318
0.4138	4.4482
0.5172	4.7243
0.6207	4.8406
0.7241	4.8784
0.8276	4.9102
0.9310	5.2830
1.0345	5.3545
1.1379	5.6890
1.2414	5.8557
1.3448	6.1572
1.4483	6.2358
1.5517	6.3070
1.6552	6.6102
1.7586	6.6784
1.8621	6.9911
1.9655	7.2086
2.0690	7.4408
2.1724	7.6436
2.2759	8.0268
2.3793	7.9795
2.4828	8.4671
2.5862	8.7794
2.6897	9.0369
2.7931	9.3041
2.8966	9.5418
3.0000	9.9061

Set up a least square problem

$$\min \|Ax - b\|_2^2$$

where $A \in R^{m \times 2}$ and $b \in R^m$ and solve it using the QR-decomposition.

Plot on the same screen the least square solution function together with data points.

2. (30 points)

(a) Suppose we want to fit the data (t_i, b_i) , $i = 1, \dots, m$, using a function

$$b(t) = x_1\varphi_1(t) + x_2\varphi_2(t) + \dots + x_n\varphi_n(t).$$

We want to find $x_1, \dots, x_n$ by solving

$$\min_x \sum_{i=1}^m w_i (b_i - x_1\varphi_1(t_i) - x_2\varphi_2(t_i) - \dots - x_n\varphi_n(t_i))^2,$$

where $w_i > 0$ is a weight reflecting the importance of the data point. For example, if the error in the i th data point is approximately e_i , then $w_i = 1/e_i$.

Formulate this problem as a linear least squares problem

$$\min_x \|\tilde{A}x - \tilde{b}\|_2^2.$$

What are $\tilde{A}$ and $\tilde{b}$?

(b) Consider the following data

t	b	error bound
0.00	20.00	20.00
0.25	51.58	24.13
0.50	68.73	26.50
0.75	75.46	27.13
1.00	74.36	26.00
1.25	67.09	23.13
1.50	54.73	18.50
1.75	37.98	12.13
2.00	17.28	04.00

Notice that the error bound varies with each data point.

Fit the data by a quadratic polynomial using ordinary least squares method. Fit the data by a quadratic polynomial using the weighting technique discussed in (a).

In each case, print out the coefficients and plot the data as well as the computed quadratic polynomial.