

Assignment 3

- (10 points) Using LU decomposition with pivoting, i.e. using $[P, L, U] = lu(A)$, derive a formula for computing the determinate of A .
- (10 points) Write your own m-function for LU -decomposition of a tri-diagonal matrix

$$A = \begin{pmatrix} d_1 & e_1 & & & \\ c_1 & d_2 & e_2 & & \\ & \ddots & \ddots & \ddots & \\ & & c_{n-2} & d_{n-1} & e_{n-1} \\ & & & c_{n-1} & d_n \end{pmatrix}.$$

Your function should take as an input vectors

$$[e_1; \dots; e_{n-1}], \quad [d_1; \dots; d_n], \quad [c_1; \dots; c_{n-1}],$$

and output vectors

$$[l_1; \dots; l_{n-1}], \quad [u_1; \dots; u_n], \quad [g_1; \dots; g_{n-1}],$$

such that

$$\begin{pmatrix} d_1 & e_1 & & & \\ c_1 & d_2 & e_2 & & \\ & \ddots & \ddots & \ddots & \\ & & c_{n-2} & d_{n-1} & e_{n-1} \\ & & & c_{n-1} & d_n \end{pmatrix} = \begin{pmatrix} 1 & & & & \\ l_1 & 1 & & & \\ & \ddots & \ddots & & \\ & & l_{n-2} & 1 & \\ & & & l_{n-1} & 1 \end{pmatrix} \begin{pmatrix} u_1 & g_1 & & & \\ & u_2 & g_2 & & \\ & & \ddots & \ddots & \\ & & & u_{n-1} & g_{n-1} \\ & & & & u_n \end{pmatrix}.$$

- (10 points) Write your own MATLAB m-function that solves the bi-diagonal systems $Ax = b$ and $Bx = b$, where

$$A = \begin{pmatrix} d_1 & & & & \\ c_1 & d_2 & & & \\ & \ddots & \ddots & & \\ & & c_{n-2} & d_{n-1} & \\ & & & c_{n-1} & d_n \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} d_1 & e_1 & & & \\ & d_2 & e_2 & & \\ & & \ddots & \ddots & \\ & & & d_{n-1} & e_{n-1} \\ & & & & d_n \end{pmatrix}$$

by back and front substitutions. Your functions should take as an input vectors

$$[e_1; \dots; e_{n-1}], \quad [d_1; \dots; d_n], \quad [b_1; \dots; b_n],$$

or

$$[c_1; \dots; c_{n-1}], \quad [d_1; \dots; d_n], \quad [b_1; \dots; b_n],$$

and output vector

$$[x_1; \dots; x_n].$$

such that $Ax = b$ or $Bx = b$.

4. (20 points)

(a) Solve the BVP

$$\begin{aligned} -y''(x) + 10y'(x) + y(x) &= f(x), \quad x \in (0, 1), \\ y'(0) &= 0, \\ y(1) &= 0, \end{aligned}$$

where

$$f(x) = (\pi^2/4 + 1) \sin(\pi(1+x)/2) + 5\pi \cos(\pi(1+x)/2)$$

with the finite differences discussed in class for $n = 100$. To solve the resulting tri-diagonal linear system use your own LU decomposition for tri-diagonal systems and back and front substitutions from above. Plot on the same screen the approximate solution and the exact solution

$$y(x) = \sin(\pi(1+x)/2).$$

(b) Use MATLAB's *tic*–*toc* functions to compare the times it takes to solve tri-diagonal linear system with your own functions and with the MATLAB's `\` function. To enter matrix A use *spdiags*.