

Assignment 2

1. (10 points)

In exact arithmetic

$$x + (y + z) = (x + y) + z.$$

Show that this may be no longer true if we use floating point arithmetic. Specifically, assume a floating point number system with $\beta = 10$ and $m = 4$.

Construct floating point numbers $\bar{x}$, $\bar{y}$, $\bar{z}$ such that

$$fl(\bar{x} + fl(\bar{y} + \bar{z})) \neq fl(fl(\bar{x} + \bar{y}) + \bar{z}).$$

2. (20 points)

Show how to rewrite the following expressions to avoid catastrophic cancelation for the indicated arguments.

- (a) $\sqrt{x+1} - 1$, $x \approx 0$,
- (b) $(1 - \cos x)/\sin x$, $x \approx 0$.

3. (30 points)

Assume the earth is a sphere of radius $r = 6370km$. Then its surface area is given by $A = 4\pi r^2$.

- (a) Using four-digit floating point arithmetic ($\beta = 10$ and $m = 4$) compute the surface area of the earth.
- (b) Using the same formula and precision, compute the difference in surface area if the value for the radius is increased by $1km$.
- (c) Since

$$\frac{d}{dr}A = 8\pi r,$$

the change in surface area is approximated by $8\pi r\delta$, where δ is the change in radius. Use this approximation and the floating point arithmetic in (a) to compute the difference in surface area if the value for the radius is increased by $\delta = 1km$.

- (d) Determine which answer, (b) or (c), is more accurate.
- (e) Explain the results you obtain in (a)(c).
- (f) Redo the computations in (a)(c) on MATLAB. How small must the change δ in radius be for the same phenomenon to occur?