Section 2 Math 227 Midterm Exam October 24, 1995

Name:______________________________

Social Security Number: __________________

Before starting to work, make sure that you have a *complete* exam: **4** numbered pages stapled to this page.

Show all work clearly in the space provided. For full credit, include all steps necessary for your answer to be a clear and logical consequence of the preceding work. To be eligible for partial credit, all row-reduction work must carry clear labels indicating which operations you perform.

Budget time by allotting no more than 7 minutes for each 10 points. Do not write anything on this page below the following dashed line.

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1. ______ **4.** ______

2. ______ **5.** ______

3. ______ **6.** ______ **(Bonus)**

TOTAL SCORE: ______

1. Consider the system of linear equations

$$\begin{cases} x_1 - 3x_2 - 2x_3 = 2 \\ \quad\quad\;\; x_2 - x_3 = 1 \\ -2x_1 + 3x_2 + 7x_3 = -7 \end{cases}$$

(a) (15 points) Find the solution set of the system.

(b) (5 points) What does your work tell you about the intersection in $\mathbf{R}^3$ of the three planes $x - 3y - 2z = 0$, $y - z = 0$, and $-2x + 3y + 7z = 0$? Explain your answer!

(c) (10 points) What can you conclude about the linear independence/dependence of the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$, where $\mathbf{u} = (1, 0, -2)$, $\mathbf{v} = (-3, 1, 3)$, and $\mathbf{w} = (-2, -1, 7)$? (If independent, explain how that follows from your work. If dependent, give a nontrivial linear relation among the vectors.)

(d) (5 points) Is the coefficient matrix A of this system of equations invertible? Why, or why not?

(e) (5 points) From your work in (a), give the LU-factorization of A.

(f) (5 points) If $\mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 2 \\ 1 \\ -7 \end{bmatrix}$, then find the solution set of the system $L\mathbf{y} = \mathbf{b}$, where L is the matrix found in (e).

(g) (5 points) Without any further work, write down the solution of the system $U\mathbf{x} = \mathbf{y}$, where U is the matrix found in (e) and $\mathbf{y}$ is the vector found in (f).

2. Consider the linear transformation $T : \mathbf{R}^3 \to \mathbf{R}^3$ whose standard matrix is the matrix A of Question 1(d).

(a) (3 points) Without doing any additional computations, write down $T(\mathbf{i})$, $T(\mathbf{j})$, and $T(\mathbf{k})$.

(b) (3 points) Write the formula for $T(x, y, z)$ for (x, y, z) an arbitrary vector in $\mathbf{R}^3$.

(c) (9 points) Does T map $\mathbf{R}^3$ onto $\mathbf{R}^3$? Why or why not?

3. (10 points) Suppose that I, 0, X, Y, A, B, C, and D are square matrices of appropriate sizes so that all products are defined and $\begin{bmatrix} I & X \\ 0 & Y \end{bmatrix}\begin{bmatrix} A & B \\ C & 0 \end{bmatrix} = \begin{bmatrix} D & B \\ I & 0 \end{bmatrix}$, where 0 stands for a zero matrix and I for an identity matrix. Then express X and Y in terms of A, B, C, and D.

4. (15 points) For $P = \begin{bmatrix} 1 & 0 & 5 \\ 1 & 1 & 0 \\ 0 & 1 & -4 \end{bmatrix}$, determine whether P is invertible. If so, find P^{-1} and write P as a product of elementary matrices. If not, explain carefully why P does not have an inverse matrix.

5. (10 points) An *affine* transformation $T : \mathbf{R}^n \to \mathbf{R}^n$ is a function of the form $T(\mathbf{x}) = A\mathbf{x} + \mathbf{b}$, where A is n-by-n and $\mathbf{b} \in \mathbf{R}^n$. Show that if $\mathbf{b} \neq \mathbf{0}$ then T is not a linear transformation.

6. **Bonus Question**. (10 points maximum, partial credit possible) Attempt this problem *only* after having completed and checked over the earlier questions.

   If AB is an invertible n-by-n matrix, then must B also be invertible? If so, prove that. If not, give an example of matrices A and B such that B is not invertible but AB is.