

Show all work and justify your answers.

1. Solve the following system of equations using an augmented matrix and row operations

$$x_1 - 2x_2 - 6x_3 = -11$$

$$3x_2 + 7x_3 = -4$$

$$x_1 - 2x_2 + 4x_3 = 9$$

2. Is the vector  $\vec{b} = \begin{bmatrix} 8 \\ 5 \\ 0 \\ 2 \end{bmatrix}$  in  $\text{span} \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -4 \\ 0 \\ -3 \\ 5 \end{bmatrix} \right\}$ ? Why or why not?

3. If  $A$  is a  $3 \times 5$  matrix of coefficients with 3 pivot columns, is  $\vec{b} = \begin{bmatrix} 1 \\ 5 \\ -3 \end{bmatrix}$  in the span of the columns of  $A$ ? Why or why not?

4. Put the following matrix into reduced row echelon form:

$$\begin{bmatrix} 1 & 3 & -2 & -2 \\ 0 & 1 & -1 & 5 \\ -1 & -2 & 1 & 7 \\ 1 & 1 & 0 & -6 \end{bmatrix}$$

5.  $A$  is a  $17 \times 10$  matrix, and both  $\vec{u} \in \mathbb{R}^{17}$  and  $\vec{v} \in \mathbb{R}^{17}$  are in the column span of  $A$ . Is  $2\vec{u} - \vec{v}$  in the column span of  $A$ ? Why or why not?

6. The linear transformation  $T: \mathbb{R}^8 \rightarrow \mathbb{R}^{12}$  has the  $12 \times 8$  matrix  $A$  as its standard matrix.  $A$  has 8 pivots. Is  $T$  onto? Is  $T$  one-to-one? Explain your answers.

7.  $T: \mathbb{R}^6 \rightarrow \mathbb{R}^7$  has standard matrix  $A$ , which is row equivalent to the matrix below. Write the solution for  $T(\vec{x}) = \vec{0}$  in parametric vector form. (Caution)