EXAM I

Math227Q-01 Fall 2000

NAME:_____

- This is a 100 point exam, point totals for individual problems are as shown.
- You must show work and/or explain your reasoning to receive full credit, even for yes/no questions.
- You are NOT allowed to use a calculator.

GOOD LUCK!

1. Let $\mathbf{A} = \begin{bmatrix} 0 & 2 & -4 & 2 \\ 1 & 0 & 5 & 7 \\ 0 & 5 & -10 & 8 \\ 0 & -7 & 14 & -10 \end{bmatrix}$.

(a) (10 pts) Describe the set of all $\mathbf{b} \in \mathbb{R}^4$ for which the system $\mathbf{Ax} = \mathbf{b}$ is consistent.

(b) (5 pts) Write the solution set to the system $\mathbf{Ax} = \mathbf{0}$.

(c) (10 pts) Show that for any matrix $\mathbf{A}$, if $\mathbf{x}_1$ and $\mathbf{x}_2$ are solutions to $\mathbf{Ax} = \mathbf{0}$, then any linear combination of $\mathbf{x}_1$ and $\mathbf{x}_2$ is also a solution.

2. Let $\mathbf{A}$ be the following matrix.

$$\mathbf{A} = \begin{bmatrix} 1 & -2 & 2 \\ 3 & -4 & -1 \\ 1 & -4 & 8 \\ 0 & 2 & -7 \end{bmatrix}$$

(a) (5 pts) Reduce the matrix $\mathbf{A}$ into echelon form.

(b) (5 pts) Do the columns of $\mathbf{A}$ span $\mathbb{R}^4$?

(c) (5 pts) Are the columns of $\mathbf{A}$ linearly independent?

(d) (5 pts) Are the columns of $\mathbf{A}^T$ linearly independent?

3. Consider the transformations $T_1 : \mathbb{R}^3 \to \mathbb{R}^2$ and $T_2 : \mathbb{R}^2 \to \mathbb{R}^2$ where

$$T_1\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x + y + z \\ x - 2y + 3z \end{bmatrix}$$

and T_2 rotates all vectors by $\pi/2$ *clockwise* about the origin.

(a) (10 pts) Write the matrices for the transformations T_1 and T_2.

(b) (5 pts) Write the matrix for the transformation $T_2 \circ T_1$.

(c) (5 pts) Is T_1 onto?

(d) (10 pts) Explain why *no* linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^2$ can be one-to-one.

4. Let $\mathbf{A} = \begin{bmatrix} 1 & 0 & -4 \\ 2 & 1 & -6 \\ 0 & 3 & h \end{bmatrix}$.

(a) (10 pts) For which values of h is the matrix $\mathbf{A}$ invertible?

(b) (10 pts) Find $\mathbf{A}^{-1}$ if $h = 5$.

(c) (5 pts) Use part (b) to solve the linear system of equations

$$\begin{array}{rrrl} x_1 & & -4x_3 & = 1 \\ 2x_1 & +x_2 & -6x_3 & = 1 \\ & 3x_2 & +5x_3 & = -1 \end{array}$$