

Math 220 Group Worksheet 6

To be done in teams without books or notes.

Names: _____

1. (2.5 minutes) Fill in the blanks to complete the statement of the second-derivative test. Suppose $f : \mathbf{R}^2 \rightarrow \mathbf{R}$ has continuous second partial derivatives on an open disk containing the point $(x, y) = (a, b)$ where $\mathbf{f}'(x, y) = \nabla f(x, y) = (0, 0)$. Let

$$D = \det \begin{pmatrix} f_{xx}(a, b) & f_{xy}(a, b) \\ f_{yx}(a, b) & f_{yy}(a, b) \end{pmatrix} = f_{xx}(a, b)f_{yy}(a, b) - [f_{xy}(a, b)]^2.$$

- (a) If $D > 0$ and $f_{xx}(a, b) < 0$ then (a, b) _____
- (b) If $D > 0$ and $f_{xx}(a, b) > 0$ then (a, b) _____
- (c) If $D < 0$ then (a, b) _____
2. (7.5 minutes, 1993 Exam 2) Find and classify the critical points of the function g with formula $g(x, y) = 2x^2 - xy^2 + y^2 + 2$.

3. (10 minutes, 1992 Exam 2) Consider the function $l : \mathbf{R}^3 \rightarrow \mathbf{R}$ with formula $l(x, y, z) = \frac{1}{\|x \mathbf{i} + y \mathbf{j} + z \mathbf{k}\|} = \frac{1}{\sqrt{x^2 + y^2 + z^2}}$ for $(x, y, z) \neq (0, 0, 0)$.

(a) Find $\frac{\partial l}{\partial x}$, $\frac{\partial l}{\partial y}$, and $\frac{\partial l}{\partial z}$.

(b) Find $\frac{\partial^2 l}{\partial x^2}$, $\frac{\partial^2 l}{\partial y^2}$, and $\frac{\partial^2 l}{\partial z^2}$.

(c) Show that l satisfies the 3-dimensional Laplace equation: $\frac{\partial^2 l}{\partial x^2} + \frac{\partial^2 l}{\partial y^2} + \frac{\partial^2 l}{\partial z^2} = 0$.