Math 220 Group Worksheet 5

To be done in teams without books or notes.

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1. (1997 Exam 1) Consider the function $f : \mathbf{R}^2 \to \mathbf{R}$ with formula $f(x, y) = \left(x^2 + y^2\right)/4$.

   (a) (2.5 minutes) Find the equation of the tangent plane to the graph of f at the point $(1, -1, 1/2)$.

   (b) (2.5 minutes) What point on the tangent plane has $x = 0.98$ and $y = -1.04$?

   (c) (2.5 minutes) Is f differentiable at $(1, -1)$? Explain!

2. (1996, Exam 1) Consider the function $g : \mathbf{R}^3 \to \mathbf{R}$ with formula $g(x, y, z) = x^2 - y^2 + z$.

   (a) (5 minutes) Find the tangent plane to the level surface $g(x, y, z) = 0$ in (a) at the point $(2, 1, -3)$.

   (b) (2.5 minutes) Find a unit normal vector to the graph of $g(x, y, z) = 0$ in (a) at the point $(2, 1, -3)$.

3. (5 minutes, 1995 Exam 1) Find formulas for $\dfrac{\partial f}{\partial x}$ and $\dfrac{\partial f}{\partial y}$ if $f(x, y) = x^2 y^3 \sin xy$.

4. (5 minutes, 1997 Exam 1) A space probe has mass $m = 20$ kg, with maximum possible error of 0.03 kg. At a certain instant, the magnitude of its acceleration is reported (to a control facility full of staff confused by metric units) to be 10.0 m/s/s with maximum error 0.05 m/s^2. Estimate the maximum error in the computed magnitude $F = ma = 20$ newtons of the force on the object at that time. (Do *not* convert anything to pounds, feet, foot-lbs, slugs, etc.!)

5. (5 minutes) Consider the function $h : \mathbf{R}^2 \to R$ with formula

$$h(x, y) = \begin{cases} \dfrac{xy}{x^2 + y^2} & \text{for } (x, y) \neq (0, 0) \\ \dfrac{1}{2} & \text{for } (x, y) = (0, 0) \end{cases}.$$

Is h differentiable at $(0, 0)$? Why or why not? (*Hint*: What can you say about the continuity of h at the origin?)