Math 220 Worksheet 2

To be done in teams without books or notes.

Names: _____

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1. (15 minutes, 1998 Exam 1) Consider the parametric curve $\mathbf{x} = \mathbf{f}(t) = \cos 3t\,\mathbf{i} + \sin 3t\,\mathbf{j} + 4t\,\mathbf{k}, t \in [0, 2\,\pi]$.

   (a) Find formulas for $\mathbf{v}(t)$, $\mathbf{a}(t)$ and $\mathbf{T}(t)$ for any value of t.

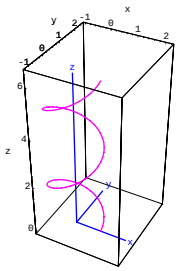
   (b) What is the arc length of the curve? Give a formula for the arc-length function $s = s(t)$ and then parametrize the curve by arc length. Verify that your arc-length parametrization is correct.

   (c) Find the unit tangent vector $\mathbf{T}$ to the path at $t = \pi/4$. Resolve $\mathbf{a}(\pi/4)$ into its tangential and normal components $\mathbf{a_T}$ and $\mathbf{a_N}$. What is the unit normal vector $\mathbf{N}$ at $t = \pi/2$?

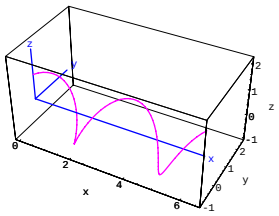
   (d) Determine the curvature at the point where $t = \pi/4$.

2. (2.5 minutes, 1999 Exam 1) Consider the parametric curve $\mathbf{x} = \mathbf{f}(t) = t\,\mathbf{i} + \sin 2t\,\mathbf{j} + \cos 2t\,\mathbf{k}, t \in [0, 2\,\pi]$. Which of the following plots is the graph of the curve? Explain how you selected it.

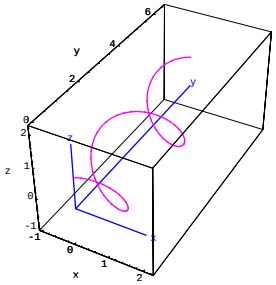
Plot A



Plot B



Plot C



Correct Plot: _____ **Reasoning:**

3. (5 minutes, 1993 Exam 1) The velocity of a moving body in the plane at any time t is $\mathbf{v}(t) = (at + 1)\,\mathbf{i} - 2b\,\mathbf{j}$, where a and b are constants. When $t = 0$ the body is at the origin. Determine the vector parametric equation of the body's path, and name the corresponding curve.