Math 220 Final Exam Review Worksheet

1. (10 minutes) If $\mathbf{v} = \mathbf{i} + 2\,\mathbf{j} + 2\,\mathbf{k}$ and $\mathbf{w} = -2\,\mathbf{i} + \mathbf{j} + 2\,\mathbf{k}$, then find
   (a) $\mathbf{v} + \mathbf{w}$ (b) $\mathbf{v} \cdot \mathbf{w}$ (c) $||\mathbf{v}||$ and $||\mathbf{w}||$
   (d) the angle between $\mathbf{v}$ and $\mathbf{w}$ (e) the coordinate of $\mathbf{v}$ in the direction of $\mathbf{w}$
   (f) the component of $\mathbf{v}$ in the direction of $\mathbf{w}$ (g) a vector perpendicular to both $\mathbf{v}$ and $\mathbf{w}$
   (h) a scalar equation of the plane through $P(1, 2, 3)$, $Q(2, 4, 5)$, and $R(-1, 3, 5)$.
2. (20 minutes) A particle moves on the curve $\mathbf{x}(t) = (\ln t)\,\mathbf{i} + 2t\,\mathbf{j} + t^2\,\mathbf{k}$, where $t > 0$. Determine
   (a) the velocity, speed, and acceleration (b) the arc length between $t = 1$ to $t = 2$
   (c) the tangential and normal components of acceleration at $t = 1$.
   (d) the unit tangent vector and unit normal vector at $t = 1$.
3. (10 minutes) Let $w = \ln(x^2 + y^2 + z^2)$, where $x = \sin st$, $y = \cos st$, and $z = s^2t^2$. Find $\partial w/\partial s$ and $\partial w/\partial t$ in terms of s and t.
4. (10 minutes) Find and classify the critical points of f if $f(x, y) = 12x + 27y - x^3 - y^3$.
5. (7.5 minutes) Find the volume of the region above the xy-plane, below the graph of $z = x^2 + y^2$, and between the graphs of $y = x^2$ and $x = y^2$.
6. (5 minutes) Evaluate $\iint_D e^{-x^2-y^2}\,dA$, where D is the region inside the unit circle $x^2 + y^2 = 1$.
7. (5 minutes) Evaluate $\displaystyle\iiint_D \frac{1}{x^2 + y^2 + z^2}\,dV$, where D is the region between the graphs of $x^2+y^2+z^2 = 1$, $x^2 + y^2 + z^2 = 9$, and $z^2 = 3x^2 + 3y^2$.
8. (7.5 minutes) Use double integration to find the area of the region D in the first quadrant between the y-axis and the graphs of $y = x$ and $y = 2 - x^2$.
9. (5 minutes) A force field $\mathbf{F}$ moves a particle from $(0, -4)$ to $(0, 4)$ in the counterclockwise direction along the ellipse $x^2/9 + y^2/16 = 1$. Calculate the work down by $\mathbf{F}$ if $\mathbf{F}(x, y) = x\,\mathbf{i} + y\,\mathbf{j}$.
10. (7.5 minutes) Evaluate $\oint_C \mathbf{F} \cdot d\mathbf{x}$, where $\mathbf{F}(x, y) = (x^3 + y^2 + y\cos x)\,\mathbf{i} + (2xy + \sin x)\,\mathbf{j}$ and C is *any* parametric smooth simple closed curve that is traversed counterclockwise as t increases. Explain your reasoning!
11. (10 minutes) If $\mathbf{F}(x, y) = x^2y^2\,\mathbf{i} + (xy^2 - x^2y^2)\,\mathbf{j}$, then is $\mathbf{F}$ conservative on $\mathbf{R}^2$? Why or why not? $\iint_S \mathbf{F} \cdot d\mathbf{S}$.