Math 220 Final Exam Review Worksheet Answers

1. (a) $-\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$ (b) 4 (c) $||\mathbf{v}|| = ||\mathbf{w}|| = 3$

   (d) $\theta = \arccos(4/9) \approx 1.1102$ Rad $\approx 63.612^\circ$ (e) 4/3

   (f) $(4/9)\,\mathbf{w} = (4/9)\,(-2\,\mathbf{i} + \mathbf{j} + 2\,\mathbf{k})$ (g) $2\,\mathbf{i} - 6\,\mathbf{j} + 5\,\mathbf{k}$

   (h) $2x - 6y + 5z = 5$

2. (a) $\mathbf{v}(t) = (1/t)\,\mathbf{i} + 2\,\mathbf{j} + 2t\,\mathbf{k}$; since $t > 0$, $||\mathbf{v}(t)|| = 2t + 1/t$; $\mathbf{a}(t) = -(1/t^2)\,\mathbf{i} + 2\,\mathbf{k}$.

   (b) $3 + \ln 2 \approx .3.69315$ (c) $\mathbf{a_T}(1) = \mathbf{T}$, $\mathbf{a_N}(1) = (-4/3, -2/3, 4/3)$

   (d) $\mathbf{T} = (1/3)(\mathbf{i} + 2\,\mathbf{j} + 2\,\mathbf{k})$, $\mathbf{N} = (-1/3)(2\,\mathbf{i} + \mathbf{j} - 2\,\mathbf{k})$

3. $\dfrac{\partial w}{\partial s} = \dfrac{4s^3t^4}{1 + s^4t^4}$, $\dfrac{\partial w}{\partial t} = \dfrac{4s^4t^3}{1 + s^4t^4}$,

4. $(-2, 3)$ and $(2, -3)$: saddle points; $(2, 3)$: local maximum point; $(-2, -3)$: local minimum point

5. $V = \int_0^1 \int_{x^2}^{\sqrt{x}} (x^2 + y^2)\,dy\,dx = 6/35$

6. $\int_0^{2\pi} \int_0^1 e^{-r^2} r\,dr = \pi(1 - 1/e)$

7. $\iiint \frac{1}{x^2+y^2+z^2}\,dV = 2\int_0^{2\pi} \int_0^{\pi/6} \int_1^3 \sin\phi\,d\rho\,d\phi\,d\theta = 4\pi(2 - \sqrt{3})$

8. $A = \int_0^1 \int_x^{2-x^2}\,dy\,dx = 7/6$

9. The work $W = \int_C \mathbf{F}(\mathbf{x}(t)) \cdot d\mathbf{x} = \int_{-\pi/2}^{\pi/2} 7 \sin t \cos t\,dt = 0$

10. By Green's theorem, if D is the region that C encloses then

$$\oint_C \mathbf{F}(\mathbf{x}(t)) \cdot d\mathbf{x} = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA \;=\; 0.$$

11. The field $\mathbf{F}$ is *not* conservative on $\mathbf{R}^2$ (its Jacobian matrix is not symmetric: see Theorem 4.6, Section 7.4, p. 444).