Exam 1 **Math 220** **October 20, 1999**

Name: ______________________________

Social Security Number: ____________________

Before starting to work, make sure that you have a *complete* exam: **5** numbered pages stapled to this page.

The point value of each question appears after its number. Budget your time accordingly: *about five minutes for each ten points.* That will leave time to check your work or to attempt the bonus question. The maximum score for averaging purposes is 100, although you can earn up to 110 points because of the bonus question.

Do not write anything on this cover page below the following solid line.

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1. ______

2. ______

3. ______

4. ______

5. ______

6. ______

7. ______ **(Bonus)**

TOTAL SCORE: ______

Show all work clearly in the space provided. For full credit, solution methods must be logical and understandable, and must involve *only* techniques and results developed thus far in *this course, Math 120–121, 115–116, or 112–114.* Clearly label your answers, be sure that they give *all* the information asked for, and that they follow logically from your preceding work. Read each question carefully! Provide *clear and explicit* directions to any work outside the workspace for its question.

1. Let $\mathbf{v} = \mathbf{i} + 2\,\mathbf{j} - \mathbf{k}$ and $\mathbf{w} = 2\,\mathbf{i} + \mathbf{j} + \mathbf{k}$.

   (a) (5 points) Determine the angle between $\mathbf{v}$ and $\mathbf{w}$.

   (b) (5 points) Find the direction of $\mathbf{w}$, that is, the unit vector in the direction of $\mathbf{w}$. Give the projection of $\mathbf{v}$ onto $\mathbf{w}$, that is, the component of $\mathbf{v}$ in the direction of $\mathbf{w}$. Finally, give the coordinate of $\mathbf{v}$ in the direction of $\mathbf{w}$.

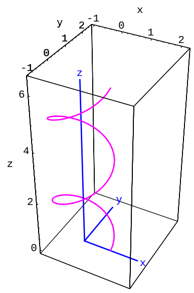
   (c) (6 points) Give vector and scalar parametric equations for the two lines l_1 and l_2 in $\mathbf{R}^3$ that pass through the point $P(1, -1, 3)$ and have the respective directions $\mathbf{v}$ and $\mathbf{w}$. Do l_1 and l_2 determine a plane in $\mathbf{R}^3$? How do you know?

   (d) (4 points) Does the line l_1 in (c) intersect the plane through $(2, -1, 3)$ with normal vector $\mathbf{n} = 2\,\mathbf{i} - \mathbf{j}$? If so, at what point? If not, how can you tell?

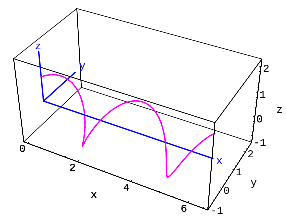
2. Consider the parametric curve $\mathbf{x} = \mathbf{f}(t) = t\,\mathbf{i} + \sin 2t\,\mathbf{j} + \cos 2t\,\mathbf{k}$, $t \in [0, 2\pi]$.

   (a) (5 points) Which of the following plots is the graph of the curve? Explain how you selected it.

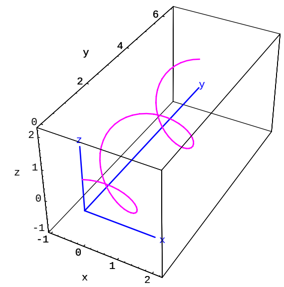
   Plot A

   Plot B

   Plot C

   Correct Plot: ________ **Reasoning:**

   (b) (5 points) Find formulas for the velocity, speed and acceleration at any time t.

   (c) (8 points) Determine the unit tangent vector and the tangential and normal components of acceleration at $t = \pi/8$.

   (d) (7 points) Find the unit normal vector $\mathbf{N}$ and the curvature K at $t = \pi/8$.

3. The graph of the $y = x^2$ in the xy-plane revolves around the y-axis.

   (a) (5 points) What is the equation of the resulting surface of revolution in $\mathbf{R}^3$?

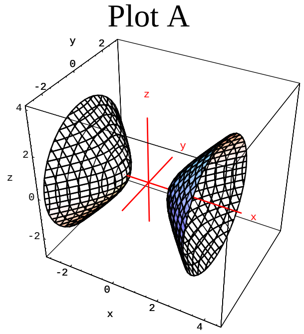
   (b) (5 points) Draw a rough sketch of the surface in the space below:

4. One of the plots below shows the quadric surface whose equation is $-x^2 + y^2 + z^2 = 1$.

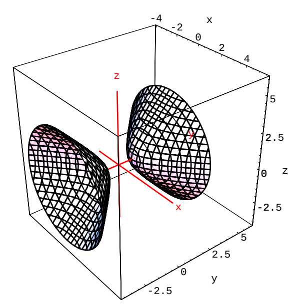
   (a) (5 points) Which of the four plots is the graph?

   Correct Plot: ______________

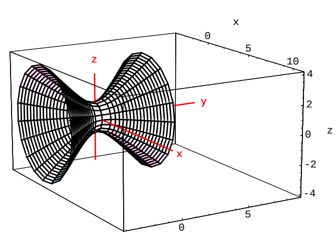
Plot A



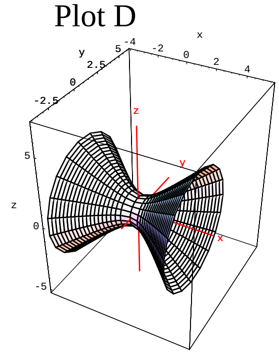
Plot B



Plot C



Plot D

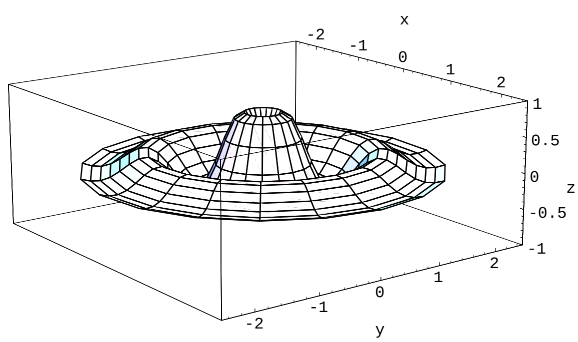


(b) (10 points) Give a complete classification and careful description of the level curves of this surface.

5. The plot below shows the graph of the *sombrero.* Its equation is

$$z = e^{-\sqrt{x^2+y^2}} \sin\left(\frac{3\pi}{2}\sqrt{x^2+y^2}\right).$$

(a) (7 points) Find the equation of the tangent plane to the sombrero at the point $(0, 1, -\frac{1}{e})$.



(b) (3 points) Use (a) to estimate the value of z when $x = 0.03$ and $y = 1.03$.

6. Recall that a circle of radius r has area $A = \pi r^2$ and circumference $C = 2\pi r$.

(a) (5 points) Express π in terms of A and C.

(b) (10 points) Suppose that the error in measuring C is e_C% and the error in A is e_A%. What is the approximate corresponding per cent error in a calculation of π from your formula in (a)?

7. **Bonus Question** (10 points maximum, partial credit possible). Attempt this problem *only* after having finished and checked over all earlier questions.

Consider the function f such that

$$f(x, y) = \begin{cases} \dfrac{xy(x^2 + y^2)}{x^4 + y^4} & \text{for } (x, y) \neq (0, 0) \\ 1 & \text{at } (x, y) = (0, 0) \end{cases} .$$

(a) Note that $f(x, y) \to 1$ as $(x, y) \to (0, 0)$ along the line $y = x$. From this, what—if anything—can you conclude about the continuity of f at $(0, 0)$? Explain!

(b) Is f in fact continuous at the origin? Why or why not?

(c) Is f differentiable at the origin? Why or why not?