

## Math 220 Exam 2 Review Worksheet

**Suggestion:** Work on this in your study groups, without books or notes, as if you were doing the problems during the weekly discussion period.

- (7.5 minutes)
  - If  $w = e^{xyz}$ , where  $x = \cos 3t$ ,  $y = \sin 3t$ , and  $z = t^2$ , then what is the numerical value of  $\left. \frac{dw}{dt} \right|_{t=\pi}$ ?
  - If  $w = \sqrt{x^2 + y^2}$ , where  $x = 2 \cos st$  and  $y = \sin st$ , then find  $\frac{\partial w}{\partial s}$ .
- (7.5 minutes) Suppose that the equation  $x^2 - y^2 - xz^3 = 1$  defines  $z$  as a differentiable function of  $x$  near the point  $(2, 1, 1)$ . Then find formulas for  $\frac{\partial z}{\partial x}$  and  $\frac{\partial z}{\partial y}$ . Give the equation of the tangent plane to the graph of the given equation at  $(2, 1, 1)$ , and estimate the value of  $z$  when  $x = 2.002$  and  $y = 0.987$ .
- (10 minutes) Consider the function  $f$  such that  $f(x, y) = x^2 - 2xy + \frac{1}{3}y^3 - 3y$ .
  - Find and classify the critical points of  $f$ .  
**Ans.:**  $(-1, -1)$ : saddle point;  $(3, 3)$ : local min. pt.
  - To find the maximum and minimum of  $f$  on the circle  $x^2 + y^2 = 9$ , what system of equation must be solved? *Do not attempt to carry out the solution of the system.*  
**Ans.:**  $x - y = \lambda x$ ,  $-2x + y^2 - 3 = 2\lambda y$ ,  $x^2 + y^2 = 9$
- (5 minutes) Set up and then evaluate the double integral that gives the area of the region between the graphs of  $y = \cos x$  and  $y = 4x^2 - \pi^2$ . **Ans.:**  $2 + \frac{2\pi^3}{3}$
- (10 minutes) Find the volume of the region in the first octant between the  $xy$ -plane, the surface  $x^2 + y^2 = 16$  and the plane  $y + z = 4$ . **Ans.:**  $16\pi - 64/3$
- (7.5 minutes) Find the volume of the solid bounded by the graphs of  $3x^2 + 3y^2 + z = 9$  and  $z = x^2 + y^2$ . **Ans.:**  $81\pi/8$
- (7.5 minutes) Consider a solid hemisphere  $H$  of radius  $a$  whose density at any point is proportional to the distance of the point from the center of the base of the hemisphere. Given that the mass of a solid is the triple integral of its density over the entire extent of the solid, find the mass of  $H$ . **Ans.:**  $k\pi a^4/2$ , where  $k$  is the constant of proportionality.
- (5 minutes) Let  $f(x, y, z) = x^2yz + e^x \cos yz$ .
  - Find the directional derivative of  $f$  in the direction of the vector  $\mathbf{v} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$  at the point  $(0, 1, \pi/4)$ .
  - What is the maximum rate of increase of  $f$  at the point given in (a)?