

Plotting Space Curves in *Mathematica*

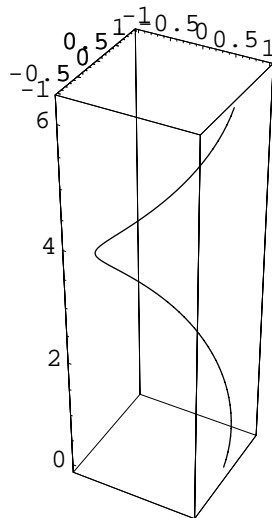
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Consider a helix in \mathbf{R}^3 that winds around the right circular cylinder perpendicular to the xy -plane with radius a . A simple parametric vector equation for the part of the curve that corresponds to $t \in [0, 2\pi]$ is

$$\mathbf{x} = a \cos t \mathbf{i} + a \sin t \mathbf{j} + t \mathbf{k}, \quad t \in [0, 2\pi].$$

The following single *Mathematica* command produces a black-and-white plot of this curve for $a = 1$. Execute the command to see that plot. (As usual, do that by placing the cursor at the end of the command and pressing the Enter key at the lower right corner of the keyboard.)

```
In[1]:= ParametricPlot3D[{Cos[t], Sin[t], t}, {t, 0, 2 Pi}]
```

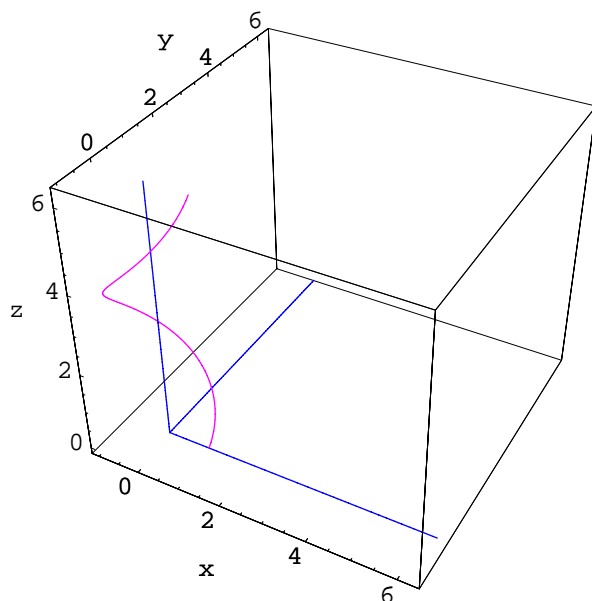


To plot the curve in *magenta* (a mix of equal parts of red and blue), use the following command.

```
ParametricPlot3D[{Cos[t], Sin[t], t, RGBColor[1, 0, 1]}, t, 0, 2 Pi]
```

The next routine adds blue coordinate axes inside the default coordinate box. It plots the axes as lines through the x -, y -, or z -axes with respective parametrizations $(t, 0, 0)$, $(0, t, 0)$, $(0, 0, t)$, for $t \in \mathbb{R}$. Since the edges of the coordinate box surrounding the curve and coordinate-axis segments carry labels, none appear along the blue coordinate axes themselves. Execute the following code to see the plot.

```
In[2]:= ParametricPlot3D[{
  {Cos[t], Sin[t], t, RGBColor[1, 0, 1]},
  {t, 0, 0, RGBColor[0, 0, 1]},
  {0, t, 0, RGBColor[0, 0, 1]},
  {0, 0, t, RGBColor[0, 0, 1]}
},
{t, 0, 2 Pi}, AxesLabel -> {x, y, z}
]
```



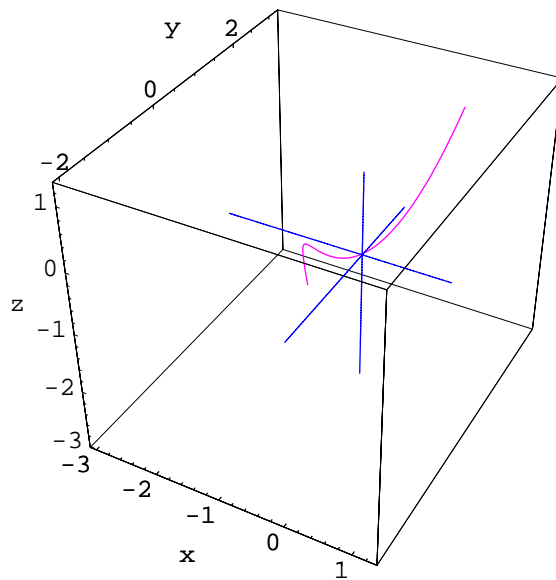
Note that the three new lines of code simply plot the three lines $\mathbf{x} = t \mathbf{i}$, $\mathbf{x} = t \mathbf{j}$, and $\mathbf{x} = t \mathbf{k}$, for t in the same range $[0, 2\pi]$ as for the helix $\mathbf{x} = \mathbf{x}(t) = \cos t \mathbf{i} + \sin t \mathbf{j} + t \mathbf{k}$.

A slight variation of the last routine plots the parametric curve

$$\mathbf{x}(t) = (t - 1)\mathbf{i} + (t^2 + 1)\mathbf{j} + (t^3 - 1)\mathbf{k}, \quad t \in [-2, 1.25]$$

from Exercise 3 of Section 2.1 of *Multivariable Calculus* by Hurley.

```
In[4]:= ParametricPlot3D[{  
  {t - 1, t^2 + 1, t^3 - 1, RGBColor[1, 0, 1]},  
  {t, 0, 0, RGBColor[0, 0, 1]},  
  {0, t, 0, RGBColor[0, 0, 1]},  
  {0, 0, t, RGBColor[0, 0, 1]}  
}, {t, -2, 1.25}, AxesLabel -> {x, y, z} ]
```



Slight editing of that produces the following routine, which plots the parametric curve in Exercise 6 of Section 2.1: $\mathbf{x}(t) = t \sin t \mathbf{i} + 3t \mathbf{j} + t \cos t \mathbf{k}$, for $t \in [-\pi, \pi]$.

```
In[5]:= ParametricPlot3D[{
  {t*Sin[t], 3*t, t*Cos[t], RGBColor[1, 0, 1]},
  {t, 0, 0, RGBColor[0, 0, 1]},
  {0, 2*t, 0, RGBColor[0, 0, 1]},
  {0, 0, t, RGBColor[0, 0, 1]}
], {t, -Pi, Pi}, AxesLabel -> {x, y, z}
]
```

