Math 116/04 Group Term Project

Due April 18, 2002

Working with your group, prepare a report on the following. List all the members of your group, each of whom should contribute in a substantial way to the report.

Provide a coherent, concise discussion of the techniques you employ, the results they produce, and the conclusions that you draw from that work. Clearly describe how you obtained any computer output that you include, and explain its significance. You do not need to prepare your report with a word processor, but neatness is necessary for an optimal score. Maximum score: 50 points.

Exercise 31 of §12.5 (p. 765) uses material that we skipped from §§12.1 and 12.3 to establish the identity

$$\ln 2 = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}. \tag{1}$$

1. Let $I_n = \int_0^{\pi/4} \tan^n x\, dx$.

   (a) Use a suitable change of variable to show that

   $$I_n = \int_0^1 \frac{u^n}{1+u^2}\, du. \tag{2}$$

   (b) Show that $I_1 = \frac{1}{2}\ln 2$.

   (c) If $0 < u < 1$, then explain why $2u < 1 + u^2 < 2$.

   (d) Use part (c) and (2) to show that for all natural numbers n,

   $$\frac{1}{2(n+1)} < I_n < \frac{1}{2n}.$$

   What then is $\lim_{n\to\infty} I_n$?

2. Show that $I_n + I_{n+2} = \frac{1}{n+1}$ for every natural number n.

3. Let $J_n = (-1)^{n-1} I_{2n-1}$.

   (a) Show that $J_n - J_{n-1} = \frac{(-1)^{n-1}}{2n-2}$.

   (b) Use part (a) and part (b) of Question 1 to show that

   $$J_n = \frac{1}{2}\ln 2 - \frac{1}{2}\sum_{k=1}^{n-1} \frac{(-1)^{k-1}}{k}.$$

   (c) Finally, use the last result and part (d) of Question 1 to establish Equation (1).