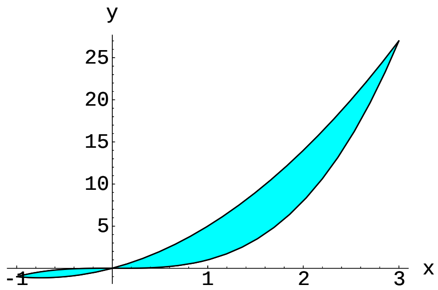
Math 116Q **Review Problems for Exam 1** **Spring, 2002**

1. Find the total area of the regions surrounded by the curves $y = x^3$ and $y = 2x^2 + 3x$.



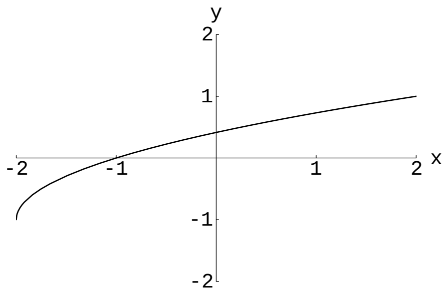
2. Find the volume of the donut that results from revolving the disk with center (3, 0) and radius 2 about the y-axis.

3. Consider the function f for which $f(x) = e^{-x} + e^{-3x}$.

   (a) Show that f is one-to-one, and determine the range of f.

   (b) Since f is one-to-one, it has an inverse function $g = f^{-1}$, whose domain is the range of f. Calculate $g'(2)$.

4. The figure shows the graph of a function f.



   (a) Explain how you know from this graph that f has an inverse function.

   (b) Draw a sketch of the graph of f^{-1} on the figure.

5. $\log_4 64 =$ ________, $\log_{27} \frac{1}{9} =$ ________

6. Solve for x: $\ln|x - 1| = \ln 4 - \ln|x + 2|$

7. If $G(x) = \displaystyle\int_{x^2}^{x^3} \frac{1}{t}\, dt$ then compute the first two derivatives of G.

8. Find (i) $\frac{dy}{dx}$ and (ii) $\int y\, dx$ if $y =$

   (a) $x^2 e^{3x}$ (b) 5^x

   (c) $x^2 \ln(x^3 + 2)$

9. In 1900, the population of Malthusville was 400,000. By 1940 the population was 1,000,000. Assume that the change was governed by the law of exponential growth.

   (a) Find a formula for the population $P(t)$, if t is the number of years since 1900.

   (b) From (a) predict the population of Malthusville in 2000.

   (c) For what value of t does the model of part (a) predict that the population will reach 5,000,000?

10. Suppose that on January 1, 1998, you invested \$1000 in the Stodgy Bond Fund, in expectation that it would double every twelve years. On January 1, 1999, suppose that you invested \$1000 in a ten-year certificate of deposit that guaranteed to double your investment in ten years. Assuming continuous compounding in both accounts, at what time would they have the same value?

11. Write *exact* (not approximate!) values for:

   (a) $\arcsin(\frac{1}{2})$ (b) $\arcsin'(\frac{1}{2})$

   (c) $\arctan(\sqrt{3})$ (d) $\arctan'(\sqrt{3})$

12. Evaluate:

   (a) $\displaystyle\int_0^{\pi/2} \frac{\sin x \cos x}{\sqrt{4 - \sin^4 x}}\, dx$ (b) $\displaystyle\int_1^{e^3} \frac{dx}{x(9 + \ln^2 x)}$

   (c) $\displaystyle\int \frac{dx}{x^2 + x + 1}$

13. Evaluate the limits:

   (a) $\displaystyle\lim_{x\to\pi/2} \cos x \cdot \tan x$ (b) $\displaystyle\lim_{x\to\pi/2} \frac{\sin x - 1}{\cos x}$ (c) $\displaystyle\lim_{x\to\infty} \frac{\ln(x^8 + 1)}{\ln(x^6 + 1)}$

   (d) $\displaystyle\lim_{x\to\infty} x\left(\frac{\pi}{2} - \arctan x\right)$ (e) $\displaystyle\lim_{x\to\infty} (1 + x)^{1/x}$ (f) $\displaystyle\lim_{x\to\frac{\pi}{2}^-} (1 + \tan x)^{\cot x}$

14. Evaluate the integrals:

   (a) $\displaystyle\int_0^{\pi/2} x^2 \cos x\, dx$ (b) $\displaystyle\int x^{10} \ln x\, dx$ (c) $\displaystyle\int_1^{e} x^2 \ln^2 x\, dx$

   (d) $\displaystyle\int x^3 e^x\, dx$ (e) $\displaystyle\int \frac{dx}{\sqrt{4 - x^2}}$ (f) $\displaystyle\int \cos^2 x\ \sin^3 x\, dx$

   (g) $\displaystyle\int \frac{dx}{x^2(x-2)(x+4)}$. (h) $\displaystyle\int \frac{2x + 3}{x^4 + x^2}\, dx$.