Math 116Q **Answers to Practice Problems for Exam 1** **Spring, 2002**

1. $A = \int_{-1}^{0}\left[x^3 - \left(2x^2 + 3x\right)\right] dx + \int_0^3\left[2x^2 + 3x - x^3\right] dx = 71/6.$
2. $V = \pi \int_{-2}^{2}\left[\left(3 + \sqrt{4 - y^2}\right)^2 - \left(3 - \sqrt{4 - y^2}\right)^2\right] dy = 24\pi^2$
3. (a) $f'(x) = -e^{-x} - 3e^{-3x} < 0$ for all x, so f is decreasing and hence (1-1). Its range is $(0, +\infty)$.

   (b) $g'(2) = \frac{1}{f'(0)} = -\frac{1}{4}.$
4. (a) From the figure, f is increasing, hence (1-1) so has an inverse.

   (b) Reflect the given graph in the line $y = x$.
5. $\log_4 64 = 3$, $\log_{27} \frac{1}{9} = -2/3.$
6. $x = 2$ or -3.
7. $G(x) = \ln x^3 - \ln x^2 = \ln x$, so $G'(x) = \frac{1}{x}$, $G''(x) = -\frac{1}{x^2}$.
8. (a) $\frac{dy}{dx} = \left(2x + 3x^2\right)e^{3x}$, $\int y\,dx = \left(\frac{1}{3}x^2 - \frac{2}{9}x + \frac{2}{27}\right) e^{3x} + C$

   (b) $\frac{dy}{dx} = 5^x \ln 5$, $\int y\,dx = 5^x / \ln 5 + C$

   (c) $\frac{dy}{dx} = 2x \ln\left(x^3 + 2\right) + 3x^4/\left(x^3 + 2\right)$, $\int y\,dx = \frac{1}{3}\left[\left(x^3 + 2\right) \ln\left(x^3 + 2\right) - x^3 - 2\right] + C$
9. (a) $P(t) = 400{,}000\, e^{kt}$, where $k = [\ln(5/2)]/40$.

   (b) $400{,}000\, e^{2.5 \ln 2.5} \approx 3{,}953{,}000$.

   (c) $t = [40 \ln(25/2)]/\ln(5/2) \approx 110.3$
10. $1000e^{t \ln 2/12} = 1000e^{(t-1) \ln 2/10} \Rightarrow t = 6.$
11.

    (a) $\arcsin(\frac{1}{2}) = \frac{\pi}{6}$ (b) $\arcsin'(\frac{1}{2}) = \frac{2}{\sqrt{3}}$

    (c) $\arctan\left(\sqrt{3}\right) = \frac{\pi}{3}$ (d) $\arctan'\left(\sqrt{3}\right) = \frac{1}{4}$
12. Evaluate:

    (a) $\frac{\pi}{12}$ (b) $\frac{\pi}{12}$ (c) $\frac{2}{\sqrt{3}} \arctan\left(\frac{2x+1}{\sqrt{3}}\right) + C$
13. Evaluate the limits:

    (a) 1 (b) 0 (c) $\frac{4}{3}$

    (d) 1 (e) 1 (f) 1
14. Evaluate the integrals:

    (a) $\dfrac{\pi^2}{4} - 2$ (b) $\dfrac{x^{11}}{11} \ln x - \dfrac{x^{11}}{121} + C$ (c) $\dfrac{5e^3 - 2}{27}$

    (d) $\left(x^3 - 3x^2 + 6x - 6\right) + C$ (e) $\arcsin(x/2) + C$ (f) $-\frac{\cos^3 x}{3} + \frac{\cos^5 x}{5} + C$

    (g) $-\dfrac{9}{32} \ln|x| - \dfrac{1}{8x} + \dfrac{1}{24} \ln|x - 2| - \dfrac{1}{96} \ln|x + 4| + C$

    (h) $2 \ln|x| - \dfrac{3}{x} - \ln\left(x^2 + 1\right) - 3 \arctan x + C$