Mathematics 116Q **Saturday 13 May 2000** **3:30-5:30 pm**

FINAL EXAMINATION

Name: ______________________________

Social Security number: ____________________ **Section:** __________

Instructor's name: ______________________________

Before starting to work, make sure that you have a *complete* exam: 8 numbered pages including this one.

The point value of each question is indicated after its statement. Budget your time accordingly: *about five minutes for each ten points*. That will leave time to check your work. There are 175 points and an additional 20 bonus points.

No books or other references are permitted. Calculators are permitted, but they must not be able to perform symbolic operations or store symbolic formulas.

Show all work clearly in the space provided. For full credit, solution methods must be complete, logical and understandable, and must involve *only* techniques and results developed thus far in *this course* (and its prerequisites). Answers must be *clearly labeled*, must give the information asked for, and must follow logically from earlier work. *Be sure to read the question carefully!* Work done outside the question's space can be considered *only* if there are clear and explicit directions to it within the question's workspace. Mark out (or fully erase) any work that you do not want graded.

Assume as known the basic facts on geometric series, p-series, and the Maclaurin series (Taylor series about 0) of e^x, $\sin x$ and $\cos x$. That is, you may quote these facts without justifying them.

Do not write anything on this cover page below the following solid line.

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1. __________	**10.** __________
2. __________	**11.** __________
3. __________	**12** __________
4. __________	**13.** __________
5. __________	**14.** __________
6. __________	**15.** __________
7. __________	**BONUS 1.** __________
8. __________	**BONUS 2.** __________
9. __________	

TOTAL SCORE: __________

1. (10 points) Compute the derivative: $\dfrac{d}{dx}(\ln 2x + 5^{\tan x})$.

2. (8 points) The graph of the function f and one of its tangent lines is depicted below. f has an inverse function $g = f^{-1}$. Evaluate $g'(1)$.

3. (8 points) Find the limit of $x^{-1}\tan^{-1}(7x)$ (that is, $x^{-1}\arctan(7x)$) as x tends to 0 . Show the steps, not just an answer.

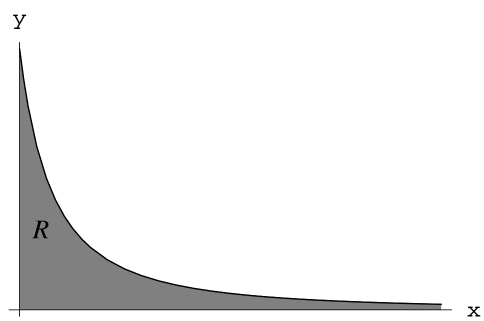
4. (14 points) Compute $\int_{1}^{\sqrt{3}} \sqrt{4-x^2}\,dx$. Show your work.

5. (13 points) Compute the volume of the solid obtained by rotating about the x-axis the region bounded by the axes, the curve $y = \sqrt{x\cos x}$, and the vertical line $x = \dfrac{\pi}{2}$. Show your work.

6. (10 points) Use **Simpson's rule** with $n = 4$ subintervals to approximate the value of $\int_{-1}^{1} \frac{1}{1+|x|}\,dx$. Your answer should be a number. Show your work.

7. (20 points) (*a*) Compute the indefinite integral $\int \frac{1}{(x+1)(x+3)}\,dx$.

(*b*) Find the area of the region in the first quadrant bounded by the axes and the curve $y = \frac{1}{(x+1)(x+3)}$. (See figure below.)



8. (10 points) Suppose that $0 < \theta < \pi/2$. Consider the infinite series

$$\sum_{n=0}^{\infty} \cos^2\theta \cdot \sin^n\theta = \cos^2\theta + \cos^2\theta \cdot \sin\theta + \cos^2\theta \cdot \sin^2\theta + \cdots.$$

Determine whether the series converges. If it does, express its sum as simply as possible; if it does not, explain why not.

9. (10 points) Determine whether the infinite series

$$\sum_{n=1}^{\infty} \frac{1}{2n^{3/4} - 1}$$

converges. Justify your conclusion.

10. (27 points) Suppose that $f(x) = \sum_{n=1}^{\infty} \frac{2^n (x+1)^n}{n}$.

(a) Find the domain of f, that is, the interval of convergence of the power series.

(b) Write the series for the derivative $f'(x)$ and find its value at $x = -\frac{4}{3}$. Your final simplified answer should be a specific number.

11. (10 points) Find the coefficient of x^{13} in the Maclaurin series (Taylor series about 0) of $f(x) = x^7 \cdot e^{-x^2}$.

12. (multiple choice, 8 points) Which of these is an equation for the tangent line to the parametric curve $x = \ln t, \quad y = e^t$ at the point where t has the value 2?

(A) $y = e^{e^x}$ (B) $y - e^2 = x - \ln 2$ (C) $y - e^2 = \frac{1}{2}e^2(x - \ln 2)$

(D) $y - e^2 = 2e^2(x - \ln 2)$ (E) $y - 2e^2 = te^t\left(x - \frac{1}{2}\right)$

(F) $y - 2e^2 = te^t(x - \ln 2)$

13. (12 points) Compute the length of the parametric curve

$$x = 3t^2, \quad y = 2t^3, \quad 0 \le t \le \sqrt{3}.$$

[*Suggestion:* Factor the expression under the radical and pull as much as you can *out* of the radical.]

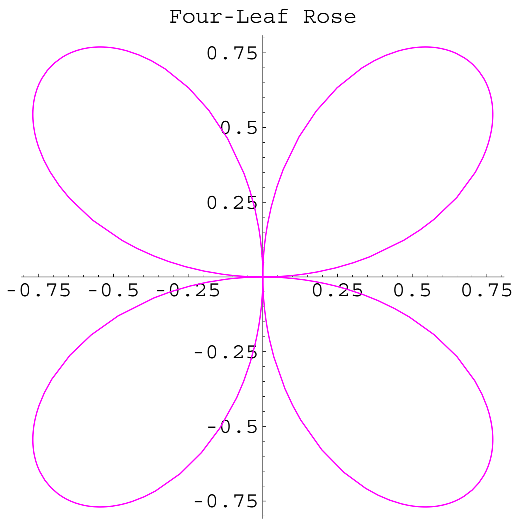
14. (multiple choice, 8 points) If a curve has polar equation $r = 8\sin\theta$, which of the following is its cartesian equation?

(A) $x^2 + (y-4)^2 = 4^2$ (B) $x^2 + (y-4)^2 = 4$ (C) $(x-4)^2 + y^2 = 4^2$

(D) $x^2 + (y-8)^2 = 0$ (E) $x^2 + (y-8)^2 = 8^2$ (F) $x^2 + y^2 = 64\sin 2\theta$

(G) $r^2 = 8r\sin\theta$

15. (7 points) Express the area enclosed by one loop of the four-leaf rose $r = \sin 2\theta$ as an integral, which you **need not** evaluate. (See figure.)



BONUS 1. (10 points) Suppose that $f(x) = \dfrac{d}{dx}\left(\dfrac{\sin 2\pi x}{e^{4x}-1}\right)$ for $x > 0$. Evaluate

$$\int_0^{1/2} f(x)\,dx.$$

BONUS 2. (10 points) f is a positive continuous function on $[0, \infty)$ that satisfies

$$\int_n^{n+1} f(x)\,dx = \frac{(\ln 5)^n}{3^n n!} \qquad \text{for} \qquad n = 0, 1, 2, 3, \ldots.$$

Evaluate

$$\int_0^{\infty} f(x)\,dx.$$

Simplify your answer as much as possible.

That's all, folks!