Mathematics 116Q **Tuesday 25 April 2000** **6:00-7:00 pm**

MIDTERM EXAMINATION 2

Name: ______________________________

Social Security number: ____________________ **Section:** __________

Instructor's name: ______________________________

Before starting to work, make sure that you have a *complete* exam: 7 numbered pages including this one.

The point value of each question is indicated after its statement. Budget your time accordingly: *about five minutes for each ten points*. That will leave time to check your work.

No books or other references are permitted. Calculators are permitted, but they must not be able to perform symbolic operations or store symbolic formulas.

Show all work clearly in the space provided. For full credit, solution methods must be complete, logical and understandable, and must involve *only* techniques and results developed thus far in *this course* (and its prerequisites). Answers must be *clearly labeled*, must give the information asked for, and must follow logically from earlier work. *Be sure to read the question carefully!* Work done outside the question's space can be considered *only* if there are clear and explicit directions to it within the question's workspace. Mark out (or fully erase) any work that you do not want graded.

Assume as known the basic facts on geometric series, p-series, and the Maclaurin series (Taylor series about 0) of $e^x; \sin x$ and $\cos x$. That is, you may quote these facts without justifying them.

Do not write anything on this cover page below the following solid line.

1.	__________	**7.**	__________
2.	__________	**8.**	__________
3.	__________	**9.**	__________
4.	__________	**10.**	__________
5.	__________	**11.**	__________
6.	__________	**BONUS.**	__________

TOTAL SCORE: __________

Name ______________________________ SS # ______________ Section ______

Math 116Q **EXAMINATION # 2** 4/25/00

1. (10 points) A function f defined on the interval $[-1{:}5; 3]$ has the following tabulated values:

x	$-1{:}5$	0	1.5	3
$f(x)$	1.2	0.5	$-0{:}2$	0.6

Estimate $\displaystyle\int_{-1.5}^{3} f(x)\,dx$ by the trapezoidal rule.

2. (10 points) (*a*) Evaluate the limit, as t approaches 0 from the right, of $\displaystyle\int_{t}^{1} \frac{1}{x^{2/5}}\,dx$.

(*b*) Write the statement about an improper integral that you have derived in part (*a*).

3. (5 points) Determine whether the **sequence** $a_1; a_2; a_3; : : : ; a_n; : : :$ (**not** the series $\sum a_n$) given by $a_n = \dfrac{\ln(n^2)}{n}$ converges or diverges, explaining fully your reasons.

4. (10 points) Does the **sequence**

$$\{1;\ 1-\frac{1}{2};\ 1-\frac{1}{2}+\frac{1}{4};\ 1-\frac{1}{2}+\frac{1}{4}-\frac{1}{8};\ 1-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+\frac{1}{16};:::\}$$

converge, and if so, to what limit? If not, why not? Use facts from the course to justify your conclusions.

5. (10 points) Determine whether the infinite **series** $\displaystyle\sum_{n=2}^{\infty}\frac{1}{\mathsf{n}[\ln \mathsf{n}]^2}$ converges or diverges, explaining how you know.

6. (5 points) Specify a number of terms of the infinite series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\mathsf{n}}$ sufficient to approximate the sum of the series to within 0.003, and state how you **know** that this number of terms is sufficient.

7. (10 points) Determine whether the infinite series $\sum_{n=1}^{\infty} (-1)^n \frac{\mathsf{n} + 1000}{\mathsf{n}^3 + 7}$ converges absolutely, converges conditionally, or diverges. Justify your conclusion.

8. (15 points) Determine the following: the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{(\mathsf{x}-2)^n}{\sqrt{\mathsf{n}}}$; the interval of numbers x for which the series converges; and for which numbers x in this interval the convergence is absolute.

9. (5 points) If

$$\mathsf{f}(\mathsf{x}) = -2\mathsf{x} + 8\mathsf{x}^3 - 18\mathsf{x}^5 + \cdots + (-1)^n 2\mathsf{n}^2\mathsf{x}^{2n-1} + \cdots = \sum_{n=1}^{\infty} (-1)^n 2\mathsf{n}^2\mathsf{x}^{2n-1}$$

for $-1 < \mathsf{x} < 1$, evaluate $\mathsf{f}'''(0)$.

10. (10 points) Define $f(x) = \int_0^x e^{-t^2}\,dt$ for any real number x. Use the Maclaurin series (Taylor series about 0) for e^x to find the Maclaurin series for the function $f(x)$, and determine for which numbers x this series actually converges to $f(x)$.

11. (10 points) Let $f(x) = \sqrt{x} = x^{1/2}$. Find the third-degree Taylor polynomial of $f(x)$ about 1, and **use it** to estimate $\sqrt{1.5}$.

BONUS. (10 points) If the power series $\sum_{n=0}^{\infty} \mathsf{c}_n(\mathsf{x}-2)^n$ converges when $\mathsf{x}=0$ and diverges when $\mathsf{x}=4$, what does the power series $\sum_{n=1}^{\infty} \mathsf{n}\mathsf{c}_n(\mathsf{x}-2)^{n-1}$ do when $\mathsf{x}=3$ and when $\mathsf{x}=-1$, and why?

That's all, folks!