

MIDTERM EXAMINATION 1

Name: _____

Social Security number: _____ Section: _____

Instructor's name: _____

- No books or other references are permitted. Calculators are permitted, but they **must not be able to perform symbolic operations** or store symbolic formulas.
- **Show all work clearly in the space provided.** For full credit, solution methods must be complete, logical and understandable, and must involve *only* techniques and results developed thus far in *this course* (and its prerequisites). Answers must be *clearly labeled*, must give the information asked for, and must follow logically from earlier work. *Be sure to read the question carefully!* Work done outside the question's space can be considered *only* if there are clear and explicit directions to it within the question's workspace. Mark out (or fully erase) any work that you do not want graded.

Some Trigonometric Identities:

$\sin^2 x + \cos^2 x = 1$	$\tan^2 x + 1 = \sec^2 x$	$1 + \cot^2 x = \csc^2 x$
$\cos 2x = \cos^2 x - \sin^2 x$	$\cos 2x = 1 - 2 \sin^2 x$	$\cos 2x = 2 \cos^2 x - 1$
$\sin 2x = 2 \sin x \cos x$	$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$	$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$

1. (10 pts.) Using calculus (no tables of values), evaluate each of the following limits:

a) $\lim_{x \rightarrow 0} \frac{\sin^{-1} x}{\sin(5\pi x)}$

b) $\lim_{x \rightarrow \infty} \frac{(\ln x)^2}{x}$

2. (10 pts.) Suppose that f is a differentiable function whose derivative is always strictly negative. The point $(3, 5)$ is on the graph of f , $f'(3) = -2$ and $f'(5) = -1$.

a) Explain how you know that f has an inverse function.

b) Let $g = f^{-1}$ be the inverse function of f . Compute $g'(5)$.

3. (10 pts.) Find the volume of the solid S obtained by revolving about the line $x = 1$ the region bounded by the line $x = 1$ and the curve $y^2 = x$.

4. (10 pts.) Consider the function $f(x) = xe^{-\pi x}$. **Using** calculus, find the interval(s) over which the function f is an increasing function. Be sure to give the endpoints of the interval(s) **exactly** - no approximations.

5. (10 pts.) Calculate the following definite integral. Simplify. Leave your answer in terms of exact numbers, not decimal approximations.

$$\int_1^e x^2 \ln x \, dx$$

6. (15 pts)

a. Simplify the expression $\cos(\tan^{-1} x)$

b. Find $f'(x)$ if $f(x) = \tan^{-1}(e^{3x})$

c. Find the derivative of $g(x) = (2 + e^x)^{(x^2)}$.

Hint: What is the derivative of $\ln g(x)$?

7. (35 pts., 7 pts. each) Integrate each of the following: All solutions **must have supporting work** showing how you arrived at your answer. Definite integrals must be evaluated exactly, **no approximations**. (E.g., if an answer is $\frac{\pi}{2}$, **do not** write 1.57079632679.)

a) $\int (x^\pi + \pi^x) dx$

b) $\int_0^{\frac{1}{\sqrt{2}}} \sqrt{1 - x^2} dx$

7. continued

c) $\int \frac{e^{(\frac{2}{x})}}{x^2} dx$

d) $\int \frac{e^x}{\sqrt{1 - e^{2x}}} dx$

e) $\int \frac{x + 1}{x(x^2 + 1)} dx$

Bonus: (6 pts.) Attempt this problem only after you have finished the rest of the exam.

Evaluate the following limit:

$$\lim_{x \rightarrow 0} \frac{\int_0^x (\sqrt{1+t^2} - 1) dt}{x^3}$$