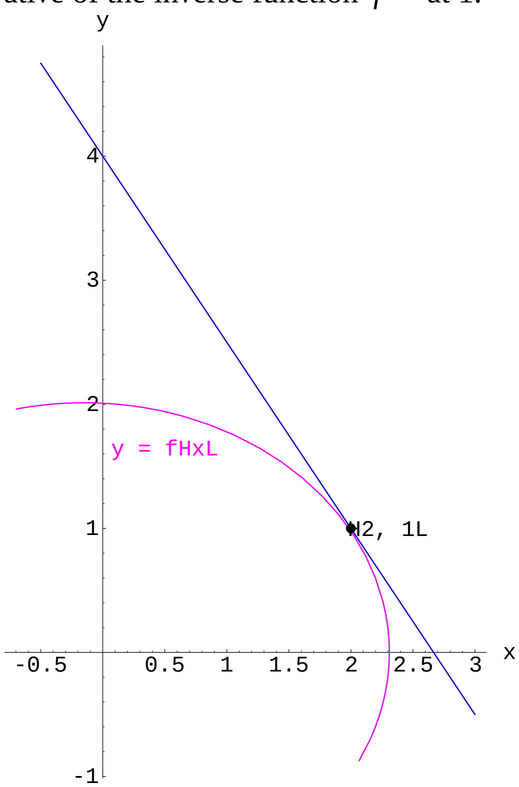
MATHEMATICS 116Q FINAL EXAMINATION

Monday 10 May 1999

Important: To maximize your score, wherever possible, espress answers exactly (in terms of sin, cos, π, e, ln, *etc.*) rather than as decimal approximations.

1. (10 points) The graph of the function f appears below. Evaluate $(f^{-1})'(1)$, the derivative of the inverse function f^{-1} at 1.



2. (10 points) Let R be the region below the curve $y = e^{-x} - \dfrac{1}{e}$ and lying in the first quadrant. Express as definite integrals (which you are **not** to evaluate):

(a) the area of R, and

(b) the volume of the solid of revolution S obtained by revolving R about the x-axis.

3. (10 points) An isotope of strontium, Sr^{90}, has a half-life of 25 years. How many years would it take for a mass of Sr^{90} to decay to 10% of the original mass?

4. (10 points) Evaluate the indefinite integral $\displaystyle\int_0^\infty re^{-r^2}\,dx$.

5. (12 points) (a) Give the *form* of the partial-fraction decomposition of $\dfrac{x^3}{(2x+1)^2(x^2+1)}$. *Do not* solve for the coefficients $A, B, C, \ldots$.

(b) Give the **form** of the indefinite integral $\displaystyle\int \frac{x^3}{(2x+1)^2(x^2+1)}\,dx$. Again, **do not** solve for the coefficients.

The next three problems are **multiple choice**. Write your selected answer as a **block capital letter** in the designated space.

6. (8 points) If $f(x) = 5^{\tan x}$ then $f'(x) =$

(A) $5^{\sec^2 x}$ (B) $\tan x \cdot 5^{\tan x - 1}$ (C) $5^{\tan x} \cdot \sec^2 x$

(D) $5^{\tan x} \cdot \ln 5 \cdot \sec^2 x$ (E) $5^{\tan x} \cdot \ln t$ (F) $5^{\sec^2 x} \cdot \ln 5$

Answer: ____________________ .

7. (8 points) The terms of a series $\displaystyle\sum_{n=1}^{\infty} a_n$ are nonzero and satisfy $a_{n+1} = \dfrac{2n-1}{3n+1} a_n$. The series then

(A) converges absolutely (B) converges conditionally (C) diverges
(D) may converge or diverge: insufficient data given to tell (E) none of these

Answer: ______________________ .

8. (8 points) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{n!} =$

(A) $\frac{1}{3}$ (B) $\frac{4}{11}$ (C) 0.367879 (D) $\frac{2}{5}$ (E) $\frac{1}{e}$

Answer: ______________________ .

9. (15 points) (a) Compute $\displaystyle\lim_{x\to\infty} x^{2/x}$

(b) Does the sequence $\left\{n^{2/n}\right\}$ converge? Why or why not? If it does, what is its limit?

(c) Does the series $\sum_{n=1}^{\infty} n^{2/n}$ converge? Why or why not?

10. (12 points) Estimate $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^4}$ to within .001 by taking the sum of the first n terms for some n, and explain how you know that you have taken enough terms to give the desired accuracy.

11. (20 points) Let

$$f(x) = \sum_{n=1}^{\infty} (-1)^n \frac{(x-1)^n}{n}.$$

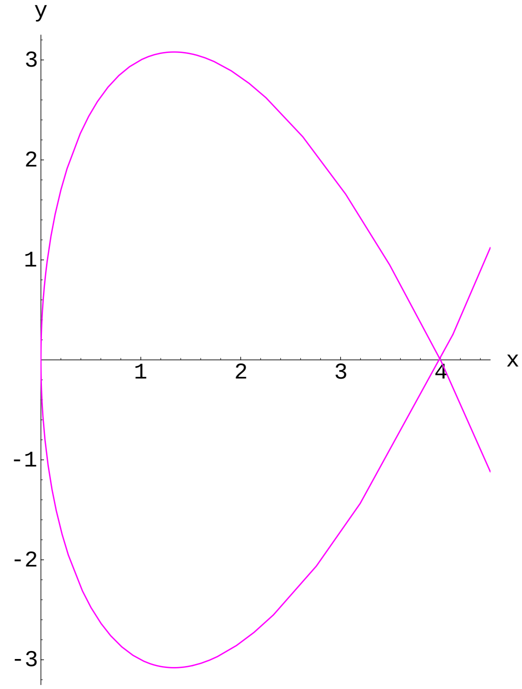
(*a*) Determine the domain of f, that is, the interval of convergence of the power series, showing how you reach your answer.

(*b*) Evaluate $f'(3/2)$, **not** $f(3/2)$.

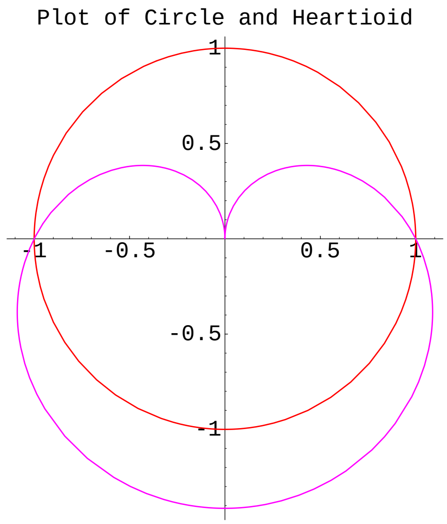
12. (10 points) Give parametric equations for the circle of radius 2 about the origin so that the circle is traversed **clockwise** as the parameter increases.

13. (10 points) Produce (but do **not** evaluate) a definite integral that gives the length of the curve $y = 2\sqrt{x}$ over the interval $1 \le x \le 3$.

14. (12 points) Consider the curve given parametrically by $x = t^2$ and $y = t^3 - 4t$. (See diagram below.) Express the length of the loop as a definite integral (which you are **not** to evaluate).



15. (10 points) The accompanying figure shows the graph of the two polar-coordinate equations $r = \sqrt{1 - \sin\theta}$ (the "heartioid") and $r = 1$. Determine the area of the region that lies inside the heartioid but outside the circle.



16. (10 points) If the vectors **v** and **w** are $\mathbf{v} =< 2, -1, 4 >$ and $\mathbf{w} =< -2, 2, 3 >$, then find a unit vector (that is, vector of length 1) parallel to (that is, in the direction of) the vector $2\mathbf{v} - 3\mathbf{w}$.

Do **at most one** of the two bonus problems. If you try both, indicate **clearly** which one you wish to have evaluated.

BONUS 1. (points) f is a positive decreasing continuous function on $[0, \infty)$ that satisfies

$$\int_{n-1}^{n} f(x)\,dx = \frac{3^n}{5^n} \qquad \text{for} \quad n = 1, 2, 3, \ldots .$$

Evaluate

$$\int_{0}^{\infty} f(x)\,dx.$$

BONUS 2. (points) There is one three-dimensional vector **w** that has first component 1 and makes angle of $\frac{\pi}{4}$ with the vector $\mathbf{u} =< 0, 1, -1 >$ and $\frac{\pi}{4}$ with the vector $\mathbf{v} =< 1, 2, 2 >$. Find the second component of **w** .

Have a great summer!