Midterm Exam 2 **Math 116** **April 20, 1999**

Name: ____________________

Social Security Number: ____________________ **Section:** ____________________

Instructor's Name: ____________________

Before starting to work, make sure that you have a *complete* exam: there are **4** numbered pages in this Web version.

The point value of each question appears after its statement. Budget your time accordingly: *about five minutes for each ten points.* That will leave time to check your work or to attempt the bonus question. The maximum score for averaging purposes is 100, although up to 110 points can be earned if the bonus is done correctly.

For full credit, solution methods must be complete, logical and understandable, and must involve *only* techniques and results developed thus far in *this course* or Math 115. Answers must be *clearly labeled*, must give the information asked for, and must follow logically from earlier work. *Be sure to read each question carefully*!

Show all work clearly in the space provided. Work done outside a question's space can be considered *only* if there are clear and explicit directions to it within the workspace. Mark out (or fully erase) any work that you do not want graded.

The absolute deadline for handing in the exam is *55 minutes after the exam begins.*

Do not write anything on this cover page below the following solid line.

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1. ______ **6.** ______

2. ______ **7.** ______

3. ______ **8.** ______

4. ______ **9.** ______

5. ______ **(Bonus)** ______

TOTAL SCORE: ______

1. (5 pts.) A function f defined on the interval $[-1.5, 3]$ has the following tabulated values:

x	-1.5	-0.75	0	0.75	1.5	2.25	3
$f(x)$	1.2	0.8	0.5	0.2	-0.2	0.3	0.6

Estimate $\displaystyle\int_{-1.5}^{3} f(x)\,dx$ by the trapezoidal rule.

2. (5 pts.) Suppose that F is an antiderivative of f, $F(5) = 7$ and $\displaystyle\lim_{x\to\infty} F(x) = 5$.
Compute $\displaystyle\int_{5}^{\infty} f(x)\,dx$.

3. (10 pts., 5 pts. each) In each of the following, determine whether the **sequence** $\{a_n\}$ converges, where

a) $a_n = \dfrac{\ln(n^2)}{n}$

b) $a_n = (-1)^n \dfrac{n}{n+1}$

4. **Multiple Choice:** (25 pts., 5 pts. each) In each of the following, you are given two questions: Indicate your choices by writing clear capital block letters in the space provided. For the first choice, indicate whether the series converges absolutely, converges conditionally or diverges. For the second, indicate **all** tests that you need to use in order to determine the convergence you indicated in the first part.

a) $\displaystyle\sum_{n=1}^{\infty} \tan^{-1} n$ where $\tan^{-1} x$ is the arctangent function.

i. (A) Absolutely Convergent (B) Conditionally Convergent (C) Divergent

Choice: ______

ii. (A) Geometric Series (B) Integral Test or p-Series (C) Test for Divergence

(D) Basic or Limit Comparison Test (E) Alternating Series Test (F) Ratio Test

Choice: ______

b) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\,[\ln(n)]^2}$

i. (A) Absolutely Convergent (B) Conditionally Convergent (C) Divergent

Choice: ______

ii. (A) Geometric Series (B) Integral Test or p-Series (C) Test for Divergence

(D) Basic or Limit Comparison Test (E) Alternating Series Test (F) Ratio Test

Choice: ______

c) $\sum_{n=1}^{\infty} (-1)^n \dfrac{1}{n \cdot \ln n}$

i. (A) Absolutely Convergent (B) Conditionally Convergent (C) Divergent

Choice: _______

ii. (A) Geometric Series (B) Integral Test or p-Series (C) Test for Divergence

(D) Basic or Limit Comparison Test (E) Alternating Series Test (F) Ratio Test

Choice: _______

d) $\sum_{n=1}^{\infty} \dfrac{1}{n^2 + 2n}$

i. (A) Absolutely Convergent (B) Conditionally Convergent (C) Divergent

Choice: _______

ii. (A) Geometric Series (B) Integral Test or p-Series (C) Test for Divergence

(D) Basic or Limit Comparison Test (E) Alternating Series Test (F) Ratio Test

Choice: _______

e) $\sum_{n=1}^{\infty} a_n$ where $a_n = \dfrac{n!}{3 \cdot 5 \cdot 7 \cdots (2n+1)}$

i. (A) Absolutely Convergent (B) Conditionally Convergent (C) Divergent

Choice: _______

ii. (A) Geometric Series (B) Integral Test or p-Series (C) Test for Divergence

(D) Basic or Limit Comparison Test (E) Alternating Series Test (F) Ratio Test

Choice: _______

5. (5pts.) How many terms in $\sum_{n=1}^{\infty} \dfrac{(-1)^n}{n^2}$ must be used to approximate the sum to within .001. Show all work.

6. (15 pts., 5 pts. each) Consider the series $\sum_{n=1}^{\infty} \dfrac{(x-2)^n}{n}$.

a) Determine the radius of convergence.

b) What is the interval of convergence? Explain you answer carefully!

c) For which real numbers x is the series absolutely convergent?

7. **Multiple Choice:** (5 pts.)

If $f(x) = -2x + 4x^3 - 6x^5 + \cdots + (-1)^n 2nx^{2n-1} + \cdots$ for $x \in (-1, 1)$, then $f'''(0) =$

(A) 0 (B) 24 (C) 12 (D) -2 (E) -24

Choice: ______

8. The *error function* erf is important in statistics, engineering and science.

Its definition is

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$$

for any real number x.

(a) (10 pts.) Use the Maclaurin series for e^x (Taylor series about $x = 0$) to find the Maclaurin series for the function $\operatorname{erf}(x)$.

(b) (5 pts.) For which real numbers x does the series in (a) converge to $\operatorname{erf}(x)$? How do you know that?

9. (10 pts., 5 pts. each) Consider the curve parameterized by

$$\begin{cases} x = 3\cos\theta \\ y = 2\sin\theta \end{cases} \quad \text{where } \theta \in [0, 2\pi]$$

a) Find $\dfrac{dy}{dx}$

b) Find the equation of the tangent line to the curve at $\theta = \frac{\pi}{6}$.

Bonus: (10 pts.) Suppose that a and b are real numbers and that $c_0, c_1, c_2, \ldots, c_n, \ldots$ is a sequence of strictly positive real numbers. Suppose further that the Ratio Test was successfully used to compute the radius of convergence of the power series $\sum_{n=0}^{\infty} c_n^2 (x-a)^n$, and then the endpoints were tested, resulting in the *interval* of convergence $(-\frac{1}{9}, \frac{7}{9}]$. find the *radius* of convergence of the power series $\sum_{n=0}^{\infty} c_n^3 (x-a)^n$.