

Name: _____

Social Security Number: _____ Section: _____

Instructor's Name: _____

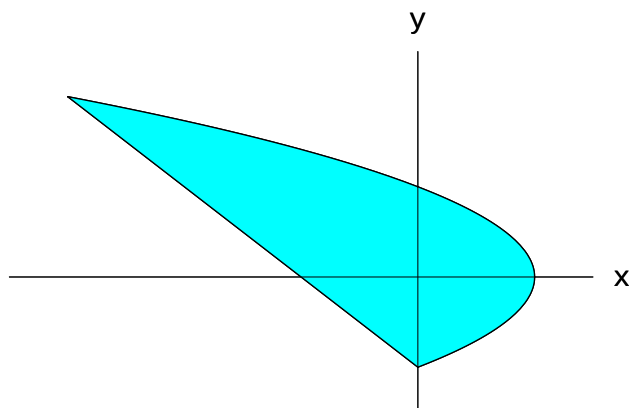
Before starting to work, make sure that you have a *complete* exam: **4** numbered pages.

The point value of each question appears after its statement. Budget your time accordingly: *about five minutes for each ten points*. That will leave time to check your work or to attempt the bonus question. The maximum score for averaging purposes is 100, although up to 110 points can be earned if the bonus is done correctly.

For full credit, solution methods must be complete, logical and understandable, and must involve *only* techniques and results developed thus far in *this course* or Math 115. Answers must be *clearly labeled*, must give the information asked for, and must follow logically from earlier work. *Be sure to read each question carefully!*

Show all work clearly in the space provided. Work done outside a question's space can be considered *only* if there are clear and explicit directions to it within the workspace. Mark out (or fully erase) any work that you do not want graded.

1. (10 points) Here is a sketch of the region bounded by the curves $x + y^2 = 1$ and $x + y + 1 = 0$. Find its area.



2. (10 points) Find the volume of the solid S obtained by revolving about the y -axis the region bounded by the y -axis and the curve $x + y^2 = 1$.

3. (10 points) Let $f(x) = x^3 + 2x$. Note that $f(1) = 3$.

(a) **Use the first derivative** of f to show that f is one-to-one.

(b) Let $g = f^{-1}$ be the inverse function of f . Compute $g'(3)$.

4. (10 points) Show that $y = f(t) = 4te^{-t^2}$ satisfies the differential equation

$$\frac{dy}{dt} + 2ty = 4e^{-t^2}.$$

5. (13 points) The population of a bacterial colony *triples* every half-hour. Its initial mass is 4 ng (nanograms).

(a) Find an expression for the mass in ng after t hours.

(b) How many hours does it take for mass of the culture to double?

6. (30 points) Evaluate the following integrals (one definite, four indefinite).

(a) $\int \frac{e^{2/x}}{x^2} dx.$

(b) $\int (x^\pi + \pi^x) dx$

(c) $\int \frac{e^x}{\sqrt{1 - e^{2x}}} dx$

(d) $\int_1^2 x^3 \ln x dx$

(e) $\int \sin^2 x \cdot \cos^3 x dx$

7. (10 points) Compute the following limits:

(a) $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2}$

(b) $\lim_{x \rightarrow 0} \frac{1 - \cos x}{(x - 1)^2}$

8. (7 points) Give the *form* of the partial-fraction decomposition of

$$f(x) = \frac{x + 1}{x^2(x^2 + 1)^2}.$$

Do not solve for the coefficients $A, B, C, \dots$

9. **BONUS.** (10 points maximum, partial credit possible). Attempt this problem *only* after having completed and checked over the earlier questions.

Evaluate

$$\lim_{x \rightarrow 0} \frac{\int_0^x (\sqrt{1 + t^2} - 1) dt}{x^3}$$