Math 5020: An Introduction to Commutative Algebra
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Assignment # 3

1: Page 32: Exercise 6

2: Page 67-72: Exercises: 3, 8, 9, 28, 30

3: Corrected Exercise 31, page 72: Let Γ be a totally ordered abelian group (written additively), and let K be a field. A *valuation of K with values in* Γ is a mapping ν: $K^* \rightarrow \Gamma$ such that:

(1) $v(xy) = v(x) + v(y)$

(2) $v(x + y) \geq \min \{v(x), v(y)\}$

for all $x, y \in K^*$. Show that the set $A = \{0\} \cup \{ x \in K^*, v(x) \geq 0\}$ is a valuation ring of K. This ring is called the *valuation ring of v,* and the subgroup $v(K^*)$ of Γ is the *value group of v*. Describe the maximal ideal of A.

4: Let A be the ring of all Gaussian integers with even imaginary parts, i.e., all $a + 2bi$, a and b integers, $i^2 = -1$. Prove that A is not integrally closed. What is the integral closure of A?