Math 5020: Introduction to Commutative Algebra
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Assignment # 2

Exercise 1: Let m and n be positive integers. Show that:

$$\mathrm{Hom}_Z(Z/mZ, Z/nZ) \cong Z/(m,n)Z$$

where Z denotes the integers, and $d = (m,n)$ denotes the greatest common divisor of m and n.

Exercise 2: Do Exercise 2 on page 31

Exercise 3*:* Let A be a commutative ring, let I and J be ideal of A, and let M be an A module. Show that: $(A/I) \otimes_A (A/J) \cong A/(I+J)$

Exercise 4: Let A be a commutative ring and let $\{M_i\}_{i \varepsilon T}$ and N be A modules. Show that $(\oplus M_i) \otimes N \cong \oplus (M_i \otimes N)$

Exercise 5: Let A be a commutative ring. Do Exercise 2.4 from the book, and conclude that any free A module is flat.

Optional addition to this exercise (only if you learned about projective modules): One definition of a projective A module is: P is a projective A module iff P is a direct summand of a free A module. Conclude that projective modules are flat.

Exercise 6: Do Exercise 5 on page 32

Exercise 7: Let G and H be Z modules (abelian groups). Determine the structure of $G \otimes_Z H$ in each of the following cases:

(i) G and H are infinite cyclic
(ii) G and H are finite cyclic
(iii) G is finite cyclic and H is infinite cyclic
(iv) G and H are finitely generated
(v) G and H are free

Exercise 8: Use Exercise 7 (ii) to do Exercise 1 on page 31. Also, find an alternative proof for Exercise 1.