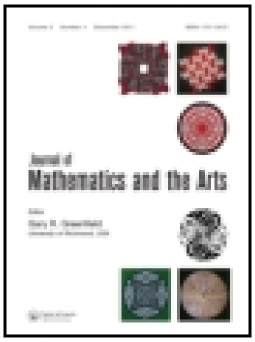

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Poetry inspired by mathematics: a brief journey through history

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This article explores one of the many manifestations of the link between mathematics and poetry—the phenomenon of poetry inspired by mathematics. Such poetry responds to the mathematical concerns and accomplishments of the day. Covering a variety of time periods and mathematical subjects, the selection of poems in this article takes the reader on a journey through the history of mathematics, highlighting along the way mathematical ideas and achievements that inspired both mathematicians and poets. We conclude with a brief discussion on the use of such poetry in the mathematics classroom.

Keywords: mathematical poetry; mathematical pedagogy; history of mathematics

AMS Subject Classifications: 01A99; 97-03; 97D99; 97U99

1. Introduction

One of the many manifestations of the intriguing link between mathematics and poetry is the phenomenon of poetry inspired by mathematics. Such poetry responds to the mathematical concerns and accomplishments of the day, be it a ground breaking definition or technique, a long standing unsolved conjecture or a celebrated theorem. The motivation for writing the poems, their mathematical subjects and their poetic styles, vary through history and from culture to culture. We bring a selection of poems from a variety of time periods, poetic styles and mathematical subjects: from a Sumerian temple hymn—where an anonymous priest counts the number of cattle in the herds of the moon god, Nanna—to contemporary poetry celebrating the proof of Fermat's Last Theorem, the still unsolved Riemann Hypothesis, or the creation of fractal geometry. Together, the poems in this article take the reader on a journey through the history of mathematics, highlighting along the way many of the mathematical ideas and achievements that inspired both mathematicians and poets. We also include references to additional sources of poetry inspired by mathematics, and conclude with a brief discussion on the use of such poetry in the mathematics classroom. This article is a considerably expanded version of the Bridges Pécs 2010 presentation [20].

2. Counting

Writing was invented in Mesopotamia, the fertile plain between the Tigris and the Euphrates rivers, situated in the region of present-day Iraq. Fragments of kiln baked clay tablets inscribed with wedge shaped cuneiform figures unearthed at archeological digs offer tantalizing glimpses of the culture, the poetry and the mathematical activities of the Sumerian, Akkadian and Babylonian civilizations that succeeded each other in the region from 4000 BC until about the second century BC. The mathematical and poetic gifts left to us by ancient civilizations seem to be intertwined. Below is an excerpt from a Sumerian temple hymn (circa 1800 BC) dedicated to the moon god, Nanna [6].

***From:* The Herds of Nanna**
By unknown author

The lord has burnished the heavens; he has embellished the night.
Nanna has burnished the heavens; he has embellished the night.
When he comes forth from the turbulent mountains, he stands as Utu stands at noon.
When Ašimbabbar comes forth from the turbulent mountains, he stands as Utu stands at noon.
. . . .
His lofty *ĝipar* shrines number four.
There are four cattle pens which he has established for him.
His great temple cattle pens, one *ece* in size, number four.
They play for him on the churn.

The cows are driven together in herds for him.
His various types of cow number 39600.
His fattened cows number 108000.
His young bulls number 126000.
The sparkling-eyed cows number 50400.

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The white cows number 126000.
The cows for the evening meal are in four groups of five each.
Such are the various types of cow of father Nanna.
………………………
Their herds of cattle are seven.
Their herdsmen are seven.
There are four of those who dwell among the cows.

They give praise to the lord, singing paeans as they move into the *ĝipar* shrines.
Nisaba has taken their grand total; Nisaba has taken their count, and she is writing it on clay.
The holy cows of Nanna, cherished by the youth Suen, be praised!
…………………………

Nisaba, appearing in the penultimate line of the hymn fragment, is the grain goddess and patroness of scribal arts and mathematical calculations. It appears that the author of this hymn needed a little divine assistance with the calculation of the grand total. This poem gives credence to the theory that one of the driving forces behind the invention of both writing and numbers, and by extension literature and mathematics, was the need to keep track of a growing quantity of riches, in particular grain and cattle.

The ancient world provides other poetic examples of what Pablo Neruda (1904–1973) called 'the thirst to know how many' [21].

***From*: Ode to Numbers**
By Pablo Neruda

Oh, the thirst to know
how many!
The hunger
to know
how many
stars in the sky!

We spent
our childhood counting
………………………

An interesting case of this phenomenon is Archimedes' *The Cattle Problem* [32]. Archimedes (287–212 BC) posed this problem in verse to the mathematicians of Alexandria in a letter he sent to Eratosthenes of Cyrene (284–192 BC). In twenty two Greek elegiac distiches (a total of 44 lines), the poem asks for the total number of cattle—white, black, dappled and brown bulls and cows, belonging to the Sun god, subject to several arithmetical restrictions. The restrictions may be divided into three sets. The first two sets of restrictions, consisting of systems of linear equations, posed some, but not insurmountable, difficulties. The problem with these sets of restrictions was posed as a challenge; it can be solved nowadays using Linear Algebra. After describing the last set of restrictions Archimedes' poem, translated into English by Hillion and Lenstra [27, 32], says.

***From:* The Cattle Problem**
By Archimedes

……………………
friend, canst thou analyse this in thy mind,
and of these masses all the measures find,
go forth in glory! be assured all deem
thy wisdom in this discipline supreme!

Attempts to solve the problem with the addition of the last set of restrictions gave rise to the *Pell Equation*, $x^2 = dy^2 + 1$, where d is an integer which is not a square, and the solutions x and y need to be positive integers. The first mathematician to solve The Cattle Problem with all the restrictions posed in the poem was A. Amthor in 1880. The solution generated a number that occupied, in reduced type, twelve journal pages—the number is approximately 7.76×10^{206544}. The Pell Equation continues to pose new 'counting difficulties' to this day, as mathematicians struggle to find efficient computer-based solution methods. Interested readers may find more information and references about The Cattle Problem and the Pell Equation in [32], and a number of more modern poems inspired by numbers and counting in [21].

3. Geometry

The British poet Samuel Taylor Coleridge (1772–1834), best known for the poem *The Rime of the Ancient Mariner*, wrote in a letter to his brother, Rev. George Coleridge, 'I have often been surprised, that Mathematics, the quintessence of Truth, should have found admirers so few . . . .'. The letter included a poem that gives an account of the proof of Proposition 1, from Book I of Euclid's (325–265 BC) *Elements*. Perhaps not quite in jest, Coleridge told his brother that the poem was a sample from a more ambitious project which intends to reproduce all of Euclid's *Elements* in a series of Pindaric odes. Unfortunately, the project was not pursued any further. Proposition 1 states that given a line segment AB, one can construct, using only a ruler and compass, an equilateral triangle with AB as one of its sides (Figure 1).

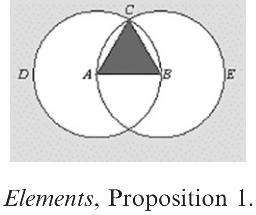


Figure 1. Euclid: *Elements*, Proposition 1.

Below is an excerpt from Coleridge's poem [42].

***From:* A Mathematical Problem**
By Samuel Taylor Coleridge

This is now—this was erst,
Proposition the first—and Problem the first.
On a given finite Line
Which must no way incline;
To describe an equi—
—lateral Tri—
—A, N, G, L, E.
Now let A. B.
Be the given line
Which must no way incline;
The great Mathematician
Makes this Requisition,
That we describe an Equi—
—lateral Tri—
—angle on it:
Aid us, Reason—aid us, Wit!

From the centre A. at the distance A. B.
Describe the circle B. C. D.
At the distance B. A. from B. the centre
The round A. C. E. to describe boldly venture.
(Third Postulate see.)
And from the point C.
In which the circles make a pother
Cutting and slashing one another,
Bid the straight lines a journeying go,
C. A., C. B. those lines will show.
To the points, which by A. B. are reckon'd,
And postulate the second
For Authority ye know.
A. B. C.
Triumphant shall be
An Equilateral Triangle,
Not Peter Pindar carp, not Zoilus can wrangle.
. . . .

Coleridge was not the only poet to be moved into verse by a beautiful geometric proof. About 100 years later Frederick Soddy (1877–1956), Nobel Prize winning British chemist, rediscovered Descartes' *Circle Theorem*—originally proved by René Descartes (1596–1650)—which involves the radii of four mutually tangent circles (Figure 2).

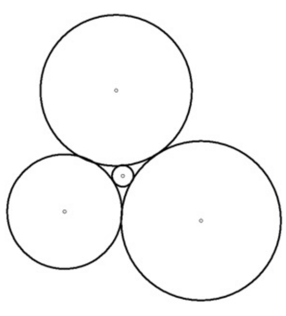


Figure 2. Four mutually tangent circles.

In his joy, Soddy wrote the verses below [21,39].

***From:* The Kiss Precise**
By Frederick Soddy

For pairs of lips to kiss maybe
Involves no trigonometry.
'Tis not so when four circles kiss
Each one the other three.
To bring this off the four must be
As three in one or one in three.
If one in three, beyond a doubt
Each gets three kisses from without.
If three in one, then is that one
Thrice kissed internally.

Four circles to the kissing come.
The smaller are the benter.
The bend is just the inverse of
The distance from the center.
Though their intrigue left Euclid dumb
There's now no need for rule of thumb.
Since zero bend's a dead straight line
And concave bends have minus sign,
The sum of the squares of all four bends
Is half the square of their sum.
. . . .

The last stanza of the poem, not included here, involves Soddy's proof of the analogous formula for spheres. After these verses appeared, Thorold Gosset (1869–1962) wrote *The Kiss Precise* (*Generalized*) (see, for example, [21]), to describe the more general case of tangency, or 'kissing', of $n+2$ hyperspheres in n dimensions. A 1980 addition to Soddy's verses is Ray Bobo's poem, *Foursomes, Fivesomes, and Orgies* [8].

Geometry followed the five postulates of Euclid's *Elements* for about 2000 years before modifications to the fifth postulate generated additional geometries. The fifth postulate states that exactly one line parallel to a given line passes through any given point, which is not on the given line (Figure 3).

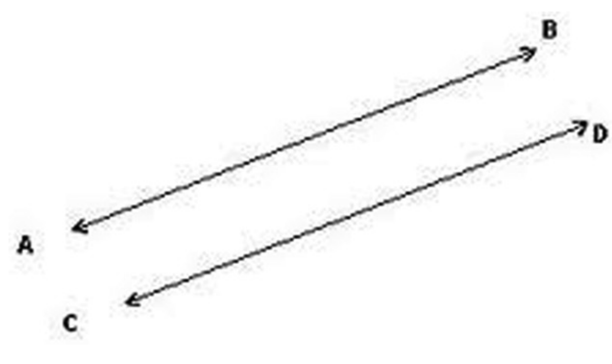


Figure 3. Parallel lines.

Eugène Guillevic's (1907–1997) poem below, translated by Richard Sieburth [26], seems to give voice to the anticipation for change.

***From*: Parallels (I)**
By Eugène Guillevic

Why stop here?
There's plenty of space
To sidle up and talk.

Though nothing you say
Could be news to the other,

You've carried on like this
From the very start.

You meet in dreams
……………

Non-Euclidean geometry was discovered independently by German mathematician Carl Friederich Gauss (1777–1855), Hungarian mathematician János Bolyai (1802–1860) and Russian mathematician Nikolai Lobachevsky (1792–1856). At that time, European thought was dominated by Kant's philosophy, which stated that Euclidean geometry *is the inevitable necessity of thought*, and none of these mathematicians were recognized for the discovery of hyperbolic geometry during his lifetime. Gauss, who disliked controversy, never published his results. Bolyai's publications roused controversy, while Lobachevsky's publications were basically ignored. Posthumously, Gauss's contribution to geometry made its way into the verse, *Ut Algebra Poesis*, by Romanian poet and mathematician Ion Barbu, aka Dan Barbilian (1895–1961) [4]; Hungarian poet Mihaly Babits (1883–1941) wrote a reverential poem in honor of Bolyai's discovery [3]; and both Gauss and Bolyai appear in Stuart Silverman's poem *K* [37]. Lobachevsky, on the other hand, seems to have gotten the short end of the poetic stick, with Tom Lehrer's funny but unflattering lyrics *Lobachevsky* [31], and the tongue-in-cheek verse in *Doorways in the Sand* by Roger Zelazny (1937–1995) [46]. Below is a poem by Michael Johnson [29,10] on an aesthetic aspect of the new geometry.

M.C. Escher's *Circle Limit III*
By Michael L. Johnson

This tessellated hyperbolic plane
is definitely non-Euclidean,
though inside, not on, its circumference
points correspond. Outside is emptiness.

The fish swim back and forth but cannot sense
how they progressively grow small or large
by distance from the unreachable edge
where hypercycles shrink to nothingness.

Thus miracled infinity is viewed,
but only by a god of finitude.

Additional poems inspired by geometry, Euclidean and otherwise, may be found in [10,21].

4. Algebra

The word *algebra* has its root in the Arabic word *al-jabru*, meaning *restoration*. In the ninth century, the Persian mathematician al-Khwarizmi (780–850) wrote an algebra book, using the word for the first time, and giving a systematic exposition of the state of knowledge regarding solutions to polynomial equations. Methods for solving linear and quadratic equations were known since Babylonian times, but the complete solution to cubic equations still awaited discovery. In 1202, the Italian mathematician Leonardo Fibonacci (1170–1240) published his book *Liber Abaci* [16], which introduced Europe to the Hindu-Arabic numeral system. The development of Algebra in Europe went hand in hand with the dissemination of this new representation of numbers. The solutions to both cubic and quartic equations were first published in *Ars Magna* (1545) by Girolamo Cardano (1501–1576), although he discovered none of the pertinent formulas. The cubic was solved by Niccolò Fontana (1500–1557), better known as Tartaglia (the 'stammerer'), while the quartic was solved by Cardano's pupil, Ludovico Ferrari (1522–1565). Cardano—physician, mathematician, gambler, autobiographer and eccentric personality par excellence—published Tartaglia's solution in his book in spite of promising Tartaglia under oath to never divulge his solution to a living soul. At that time, winning mathematical competitions was a way to make a living. These competitions were won based on the speed of solving cubic equations and methods for finding quick solutions were held in strict secrecy. This act of betrayal resulted in a lifelong feud between the two men. Far from the source of distress, we can enjoy the cryptic verse solution to the cubic given to Cardano by Tartaglia [35], even though the English translation does not preserve the *terza rima* rhyming scheme of the original Italian poem. The equations to the right, not part of the original poem, illuminate the steps outlined in the verse.

Solving the Cubic
By Niccolò Tartaglia

When the cube and the things together
Are equal to some discrete number, $[x^3 + ax = b]$
Find two other numbers differing in this one. $[u - v = b]$

Then you will keep this as a habit
That their product should always be equal
Exactly to the cube of a third of the things. $[uv + (a/3)^3]$

The remainder then as a general rule
Of their cube roots subtracted
Will be equal to your principal thing. $[x + \sqrt[3]{u} = \sqrt[3]{v}]$

In the second of these acts,
When the cube remains alone $[x^3 = ax + b]$
You will observe these other agreements:

You will at once divide the number into
two parts $[b = u + v]$
So that the one times the other produces
clearly
The cube of a third of the things exactly $[uv = (a/3)^3]$

Then of these two parts, as a habitual rule,
You will take the cube roots added together,
And this sum will be your thought. $[x = \sqrt[3]{u} + \sqrt[3]{v}]$

The third of these calculations of ours $[x^3 + b + ax]$
Is solved with the second if you take
good care,
As in their nature they are almost matched.

These things I found, and not with sluggish steps,
In the year one thousand five hundred, four
and thirty
With foundations strong and sturdy

In the city girdled by the sea. [Venice]

Progress in finding solutions to polynomial equations of degree higher than four stalled until the nineteenth century. However, methods for solving systems of linear equations using matrices and determinants, which went back to the second century BC, reappeared near the end of the seventeenth century and development of the area we call nowadays *Linear Algebra* got underway at that time. A modern Shakespearean sonnet by Michael Zahniser [45], who was a student in a *Linear Algebra* class when he wrote this poem, appears below.

A Linear Algebra Sonnet
By Michael Zahniser

This sort of math annoys a busy man,
When he has lots of precious time to kill:
"The space the vectors fill is called the span,
the span's the space the set of vectors fill.
A pivot is an isolated one
With zeroes up above and down below.
Row echelon is when your work is done,
With ohs before the ones in every row.
A transformation is a funny beast
Which maps a set of vectors to a range."
I understand this much of it at least,
And I can't help it if the rest seems strange.
Nor should I be beset by pangs of guilt,
When matrices turn out to have been tilt.

In the early nineteenth century the French mathematician Évariste Galois (1811–1832) proved that no general formula exists for solving quintic equations. Galois' result utilized, and contributed to, the development of group theory—a new area of mathematics initiated by Niels Henrik Abel (1802–1829). Galois died at the age of 20 in a duel. The night before the fatal event he is believed to have written down many of his mathematical discoveries, ideas that when fully understood—a century after Galois' death—set the foundations for modern abstract algebra. On the margins of his notes he penned the words: 'I have no time!' Below is a fragment from Carol Christopher Drake's touching poem [34] inspired by this story.

***From*: Antiphon for Évariste Galois**
By Carol Christopher Drake

Until the sun I have no time
But the flash of thought is like the sun
Sudden, absolute:
Watch at the desk
Through the window raised on the flawless dark,
The hand that trembles in the light,
Lucid, sudden.
. . . .

Algebra has developed in many different directions since the nineteenth century. A small sample of the poetry reflecting this growth, including my poem, *Late Afternoon at the Workshop on Commutative Rings*, may be found in [21]; the lyrics, *Finite Simple Group* (*of Order Two*) by the Klein Four Group, appear in [41]; while [9] contains William Bronk's (1918–1999) poem, *Boolean Algebra:* $X^2 = X$, a fragment of which is given below.

***From*: Boolean Algebra: $\mathbf{X^2 = X}$**
By William Bronk

Quantity is what is not. The truth
is ignorant of numbers. The universe
is one; is all; besides there is nothing. The square
of one, the square of nothing, is only one,
—or nothing. X^2 equals X. The truth
admits these values. What is other is not
the truth. Zero and one. An algebra
of these alone expresses everything.
. . . .

5. Calculus

Sixteenth century Europe saw a vigorous revival of mathematical activities that culminated with the invention of Calculus in the late seventeenth century—a development that marked the beginning of modern mathematics. At the heart of Calculus lies *The Fundamental Theorem of Calculus*, a seminal result that was proved independently and almost simultaneously by Isaac Newton (1643–1727) and Gottfried Leibniz (1646–1716). The towering figures of these two mathematicians, and some of the history and controversy surrounding the birth of Calculus, are captured in my poem, *Calculus* [18,21], a fragment of which appears below.

***From:* Calculus**
By Sarah Glaz

I tell my students the story of Newton versus Leibniz,
the war of symbols, lasting five generations,
between The Continent and British Isles,
involving deeply hurt sensibilities,
and grievous blows to national pride;
on such weighty issues as publication priority
and working systems of logical notation:
whether the derivative must be denoted by a "prime,"
an apostrophe atop the right hand corner of a function,
evaluated by Newton's fluxions method, $\Delta y/\Delta x$;
or by a formal quotient of differentials dy/dx,
intimating future possibilities,
terminology that guides the mind.
The genius of both men lies in grasping simplicity
out of the swirl of ideas guarded by Chaos,
becoming channels, through which her light poured clarity
on the relation binding slope of tangent line
to area of planar region lying below a curve,
The Fundamental Theorem of Calculus,
basis of modern mathematics, claims nothing more.

While Leibniz—suave, debonair, philosopher and politician,
published his proof to jubilant cheers of continental followers,
the Isles seethed unnerved,
they knew of Newton's secret files,
locked in deep secret drawers—
for fear of theft and stranger paranoid delusions,
hiding an earlier version of the same result.
……………………

Calculus—Latin for small stones,
primitive means of calculation; evolving to abaci;
later to principles of enumeration advanced by widespread use
of the Hindu-Arabic numeral system employed to this day,
as practiced by *algebristas*—barbers and bone setters in Medieval Spain;
before Calculus came the Σ (sigma) notion—
sums of infinite yet countable series;
and culminating in addition of uncountable many dimensionless line segments—
the integral $\int$—snake,
first to thirst for knowledge, at any price.
……………………………

The wonder aroused by the sum of an infinite series that is on occasion a finite number, the ability to 'add' uncountable entities, and the discovery of unexpected connections between disparate mathematical notions, inspired many other poets and mathematicians. Jacob Bernoulli (1654–1705), an important contributor to the development of Calculus, included the following verses in his posthumously published work *Ars Conjectandi* [5,21].

***From:* Treatise on Infinite Series**
By Jacob Bernoulli

Even as the finite encloses an infinite series
And in the unlimited limits appear,
So the soul of immensity dwells in minutia
And in narrowest limits no limits inhere.
What joy to discern the minute in infinity!
The vast to perceive in the small, what divinity!

Born two years after Bernoulli's death, Leonhard Euler (1707–1783), was one of the most prolific mathematicians of all time. His contributions to each of the many areas of mathematics that he became interested in are difficult to overestimate. Among other things, Euler introduced much of the modern terminology and notation that is in use today. In particular, Euler gave the number e its modern designation and discovered many of its remarkable properties, including *The Euler Identity*, which many consider to be the most elegant formula in mathematics. My poem, *The Enigmatic Number e* [22], a fragment of which appears below, touches on this theme.

From:* The Enigmatic Number *e
By Sarah Glaz

………………
In retrospect, following Euler's naming,
e lifted its black mask and showed its limit:

$$e = \lim_{x\to\infty}\left(1+\tfrac{1}{n}\right)^n$$

Bernoulli's compounded interest for an investment of one.

Its reciprocal gave Bernoulli many trials,
from gambling at the slot machines to deranged parties
where nameless gentlemen check hats with butlers at the door,
and when they leave, e's reciprocal hands each a stranger's hat.

In gratitude to Euler, e showed a serious side,
infinite sum representation:

$$e = \sum_{n=0}^{\infty}\tfrac{1}{n!} = \tfrac{1}{0!}+\tfrac{1}{1!}+\tfrac{1}{2!}+\tfrac{1}{3!}+\cdots.$$

For Euler's eyes alone, e fanned the peacock tail of
$\frac{e-1}{2}$'s continued fraction expansion,
displaying patterns that confirmed
its own irrationality.
………………

e's abstract beauty casts a glow on Euler's Identity:

$$e^{i\pi} + 1 = 0$$

the elegant, mysterious equation,
where waltzing arm in arm with i and π,
e flirts with complex numbers and roots of unity.
……………………

Poems inspired by the mathematics and the mathematicians of that period are widely spread throughout the literature. A small selection appears in [10,21].

6. Foundations

David Hilbert (1862–1943), one of the most prominent mathematicians of the twentieth century, wrote: 'The infinite! No other question has ever moved so profoundly the spirit of man,' [36].

Below is a fragment from Elizabeth Bartlett's poem [37] reflecting this sentiment.

***From:* Because I Longed**
By Elizabeth Bartlett

Because I longed
To comprehend the infinite

I drew a line
Between the known and unknown
………………………

Attempts to organize the concept of infinity mathematically appear as early as 100 BC in Indian Jain religious literature [43]. But, it was not until the end of the nineteenth century that such attempts succeeded in representing infinity with sufficient mathematical rigor to allow a deeper understanding and an ability to manipulate the concept mathematically. One man, Georg Cantor (1845–1918), is responsible for both the rigorous mathematical representation of infinity and the development of set theory—the 'language' that allowed mathematicians to work with the concept of infinity. Below are the lyrics, *Hotel Infinity* [33], by 'Mathemusician' Larry Lesser, providing a humorous way of understanding some properties of countable infinities discovered by Cantor.

Hotel Infinity
by Lawrence Mark Lesser

(*May be sung to the tune of Eagles' 'Hotel California',* 1976)

On a dark desert highway—not much scenery
Except this long hotel stretchin' far as I could see.
Neon sign in front read "No Vacancy,"
But it was late and I was tired, so I went inside to plea.

The clerk said, "No problem. Here's what can be done—
We'll move those in a room to the next higher one.
That will free up the first room and that's where you can stay."
I tried understanding that as I heard him say:

CHORUS: "Welcome to the HOTEL called INFINITY—
Where every room is full (every room is full)
Yet there's room for more.
Yeah, plenty of room at the HOTEL called INFINITY—
Move 'em down the floor (move' em down the floor)
To make room for more."

I'd just gotten settled, I'd finally unpacked
When I saw 8 more cars pull into the back.
I had to move to room 9; others moved up 8 rooms as well.
Never more will I confuse a Hilton with a Hilbert Hotel!

My mind got more twisted when I saw a bus without end
With an infinite number of riders coming up to check in.
"Relax," said the nightman. "Here's what we'll do:
Move to the double of your room number:
That frees the odd-numbered rooms."

(Repeat Chorus)

Last thing I remember at the end of my stay—
It was time to pay the bill but I had no means to pay.
The man in 19 smiled, "Your bill is on me.
20 pays mine, and so on, so you get yours for free!"

Cantor's discoveries, crossing into the domain of philosophy and religion, had profound implications for the development of mathematics. With the introduction of Non-Euclidean geometry and group theory, and the innovations in Calculus—with its infinite summation and infinitesimal quantities—mathematics was becoming increasingly more abstract and removed from the physical world. To continue advancing in this direction, there was a need for the discovery of new tools that will not only help prove theorems, but also validate their intrinsic 'truth'. Cantor's new set theory and his treatment of infinity offered the necessary tool-kit, and as such had both admirers and critics. Emily Grosholz's poem *Reflections on the Transfinite* [23] provides a glimpse into the ambivalent feelings of the opposition.

***From:* Reflections on the Transfinite**
By Emily Grosholz

Reading about the tower or great-boled tree
of ordinals, I think how Cantor grew
more wise and more insane, trying to save
his tree of Jesse from the pruning shears
and kitchen gardening of Kronecker;
though I must share the latter's feeling for
the natural numbers, those deceptively
well-ordered, step-wise creatures, which appear
transparent as they mount, but all in all
among themselves are most unknowable.

Dreaming about the cardinals, at night,
the alephs flaming like a candelabrum,
I see you in the attic of your house
…………………………………

'Mathematics may be defined as the subject in which we never know what we are talking about, nor whether what we are saying is true' [36]. This cynical view of mathematics is attributed to Bertrand Russell (1872–1970). The occasion may have been his discovery of a paradox in Cantor's set theory. A non-mathematical version of this paradox, known as *The Barber Paradox,* can be stated as follows: *If a barber shaves everyone in his town who does not shave himself, does he or does he not shave himself? If he does, he does not; if he does not, he does*. Russell's paradox undermined the foundations of mathematics. In an attempt to repair the damage, Russell published a resolution of this paradox in his joint work with Alfred North Whitehead (1861–1947), *Principia Mathematica*. This work attempts to reduce the foundations of mathematics to logic. Along with Hilbert's work on formalism, it had a major influence in promoting the axiomatic approach to mathematics that had been one of the major characteristics of the subject throughout the twentieth century. This approach was dealt a severe blow by Kurt Gödel's (1906–1978) result known as: *Gödel's Incompleteness Theorem*. Gödel's Incompleteness Theorem states that all consistent formulations of number theory include undecidable propositions [44]. Therefore, mathematics has statements that cannot be either proved or disproved. A related result that is sometimes known as Gödel's second incompleteness theorem can be formulated colloquially as follows: any formal system that is interesting enough to formulate its own consistency can prove its own consistency if and only if it is inconsistent. In particular, the consistency of the axioms of mathematics cannot be proved within the system. Below is a fragment of Hans Magnus Enzesberger's poem *Homage to Gödel* [15].

***From*: Homage to Gödel**
By Hans Magnus Enzensberger

………………
Gödel's theorem may seem, at first sight,
rather nondescript,
but please keep in mind:
Gödel is right.

"In any sufficiently rich system
statements are possible
which can neither be proved
nor refuted within the system,
unless the system itself
is inconsistent."

You can describe your own language
in your own language:
but not quite.
You can investigate your own brain
by means of your own brain:
but not quite.
Etc.

In order to be vindicated
any conceivable system
must transcend, and that means,
destroy itself.

"Sufficiently rich" or not:
Freedom from contradiction
is either a deficiency symptom,
or it amounts to a contradiction.

(Certainty = Inconsistency.)
………………

Gödel's incompleteness result is regarded as one of the most remarkable achievements of twentieth century mathematics. It had enormous impact not only on mathematics, but also on other fields such as computer science and philosophy. In fact, Gödel's work on universal formal languages and the limits of proof and computation laid the foundations of theoretical computer science. The certainty of knowing the exact extent of uncertainty inherent in the system had the beneficial effect of removing vague anxieties and redirecting focus. It did not diminish the zest with which mathematicians pursue their profession. Neither did it diminish the amazing ability of mathematics to describe the physical phenomena around us in meaningful and applicable ways.

7. Contemporary mathematics

It is easier to gain perspective on past mathematical accomplishments, than to characterize the mathematical landscape of the present. Nevertheless, it is possible to point out several contemporary mathematical results that have had an impact on the popular and mathematical cultures. Foremost among these is fractal geometry, which was developed by Benoît Mandelbrot (1924–2010) in 1970. In less than 50 years fractal geometry has become so entrenched in culture that it needs no introduction—the image of the Mandelbrot set (Figure 4) is immediately recognizable by every literate person. One can speculate whether the appeal of fractal geometry lies in the beauty of its computer generated images or in its power to describe the seemingly random and chaotic order of the world. Whatever the explanation, fractal geometry stars in many poems. The verse fragment below comes from a

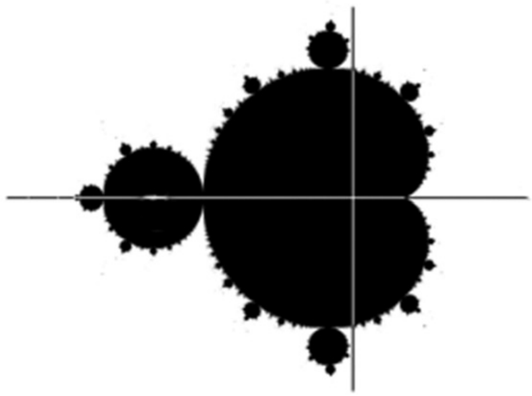


Figure 4. The Mandelbrot set.

song lyric composed by American folk-pop singer and writer Jonathan Coulton [13,21].

***from*: Mandelbrot Set**
by Jonathan Coulton

Pathological monsters! cried the terrified mathematician
Every one of them is a splinter in my eye
I hate the Peano Space and the Koch Curve
I fear the Cantor Ternary Set
And the Sierpinski Gasket makes me want to cry
And a million miles away a butterfly flapped its wings
On a cold November day a man named Benoît Mandelbrot was born

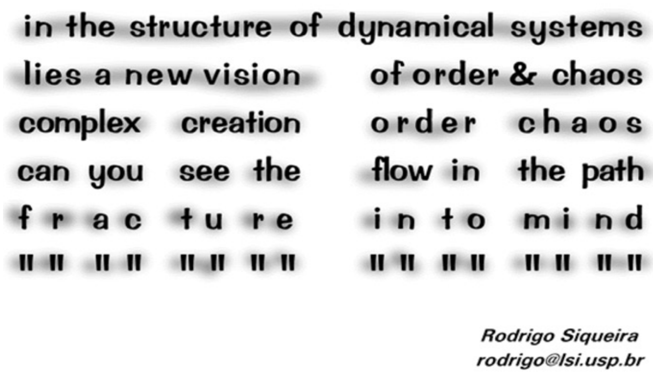
His disdain for pure mathematics and his unique geometrical insights
Left him well equipped to face those demons down
He saw that infinite complexity could be described by simple rules
He used his giant brain to turn the game around
And he looked below the storm and saw a vision in his head
A bulbous pointy form
He picked his pencil up and he wrote his secret down

Take a point called Z in the complex plane
Let Z1 be Z squared plus C
And Z2 is Z1 squared plus C
And Z3 is Z2 squared plus C and so on
If the series of Z's should always stay
Close to Z and never trend away
That point is in the Mandelbrot Set

Mandelbrot Set you're a Rorschach Test on fire
You're a day-glo pterodactyl
You're a heart-shaped box of springs and wire
You're one badass fucking fractal
And you're just in time to save the day
Sweeping all our fears away
You can change the world in a tiny way
. . . .

Rodrigo Siqueira's concrete poem, *The Cantor Dust* [38], whose image is reproduced below, is a different example of 'fractal poetry'. It uses the visual structure of the *Cantor Set* to send a poetic and mathematical message. The Cantor Set is constructed by repeatedly deleting the middle third from each interval of a growing collection of intervals. The construction starts with the deletion of the middle third from the interval [0, 1]. Then, the middle third from each of the two remaining intervals is deleted. Next follows the deletion of the middle third from each of the four remaining intervals, and so on. The process never terminates.

The Cantor Dust—A Fractal Poem



Additional poems on fractal geometry may be found in [7,10,11,21].

Several contemporary landmark mathematical results revolve around the great conjectures. In 1996 Andrew Wiles proved *Fermat's Last Theorem*, which states that the equations $x^n + y^n = z^n$ have no positive integer solutions for n larger than 2. The theorem was stated by Pierre de Fermat (1601–1665) on the margin of his copy of Diophantus' (circa 200–284) *Arithmetic*, accompanied by the famous words: 'I have discovered a truly remarkable proof which this margin is too small to contain'. The 350 years long quest for a solution and the substantial body of mathematics generated in the process made Wiles' proof one of the most remarkable mathematical results of the century. As expected, this event was celebrated with poetry (see, for example, [2,14,40]), most of which is humorous—a genre much loved by mathematicians. Below, as a small sample, are two limericks from *Fermat's Last Theorem Poetry Challenge* [40].

With an Integer Greater than 2
By Ted Munger

With an integer greater than 2
It's something one simply can't do.
If this margin were fat,

I'd show you all that,
But it's not, so the proof is on you!

We Take an Elliptic Curve E
By Jeremy Teitelbaum

We take an elliptic curve E,
Consider the points killed by 3,
This "rho" must be modular,
And by facts which are popular,
The proof of Fermat comes for free.

The Clay Mathematics Institute, founded in 1998, listed the seven most important open problems in mathematics, the *Millennium Prize Problems* [12]. One of the millennium prize problems, the *Riemann Hypothesis*, proposed by Bernhard Riemann (1826–1866), celebrated its one hundred and fiftieth anniversary in 2010. The Riemann Hypothesis is a conjecture about the zeros of the Riemann zeta function. It is considered to be the most important open problem in pure mathematics, one whose solution would advance our knowledge of the distribution of prime numbers. Below are Tom Apostol's Riemann Hypothesis lyrics [1].

Where Are the Zeros of Zeta of s?
By Tom Apostol

(*May be sung to the tune of 'Sweet Betsy from Pike'*)

Where are the zeros of zeta of s?
G.F.B. Riemann has made a good guess;
They're all on the critical line, saith he,
And their density's one over 2pi log t.

This statement of Riemann's has been like a trigger
And many good men, with vim and with vigor,
Have attempted to find, with mathematical rigor,
What happens to zeta as mod t gets bigger.

The efforts of Landau and Bohr and Cramer,
And Littlewood, Hardy and Titchmarsh are there,
In spite of their efforts and skill and finesse,
In locating the zeros there's been little success.

In 1914 G.H. Hardy did find,
An infinite number that lay on the line,
His theorem however won't rule out the case,
There might be a zero at some other place.

Let P be the function pi minus li,
The order of P is not known for x high,
If square root of x times log x we could show,
Then Riemann's conjecture would surely be so.

Related to this is another enigma,
Concerning the Lindelof function mu (sigma)
Which measures the growth in the critical strip,
On the number of zeros it gives us a grip.

But nobody knows how this function behaves,
Convexity tells us it can have no waves,
Lindelof said that the shape of its graph,
Is constant when sigma is more than one-half.

Oh, where are the zeros of zeta of s?
We must know exactly, we cannot just guess,
In order to strengthen the prime number theorem,
The integral's contour must not get too near 'em.

Several other poems were inspired by the millennium prize problems. In 2004, Grigory Perelman was offered the Fields Medal for solving the millennium prize problem called *The Poincaré Conjecture* [12]. He refused to accept the award. JoAnne Growney's poem, *Perelman and Me* [24], grapples with the complex emotions aroused by this refusal. The poetry muse was silent when in 2010 Perelman turned down the 1 million dollar Clay Millennium Prize for the resolution of the Poincaré Conjecture. A poem by Haipeng Guo, *When a P-man Loves an NP-woman* [21], involves the still unsolved millennium prize problem, *P vs. NP* [12]. In this poem, the NP-woman asks the P-man to solve the conjecture as a condition for marrying him. The Navier-Stokes equations, the subject of another one of the millennium prize problems, are featured in a poetic interpretation video prepared by Kim Lasky, Peter Childs, Abdulnaser Sayma, and their students at the Engineering School of the University of Sussex, UK [30].

8. A poem for the future

Paul Erdős (1913–1996), the most prolific and eccentric mathematician of our time, devoted his life to mathematics, which he saw as a quest for lasting beauty and ultimate truth. Erdős, living without a permanent position or home, pursued his quest for six decades and across four continents, publishing 1525 articles jointly with 511 collaborators. Regarding the old debate whether mathematics is created or just discovered, Paul Hoffman, who published a biography of Erdős, *The Man Who Loved Only Numbers* [28], says: 'I'm not qualified to say whether or not God exists', Erdős said. 'I kind of doubt He does. Nevertheless, I'm always saying that the SF [God] has this transfinite Book—transfinite being a concept in mathematics that is larger than infinite—that contains the best proofs of all mathematical theorems, proofs that are elegant and perfect'. The strongest compliment Erdős gave to a colleague's work was to say, 'It's straight from the Book'.

Sandra Gilbert's poem [17] echoes Erdős' views. Most mathematicians will agree: it is all already there, written in the *Book of Proofs*, for future generations to discover.

He Explains the Book Proof
By Sandra M. Gilbert

The shadowy clatter of the café
frames the glittering doorway.

A white cup and a blue bowl
inscribe pure shapes on the table.

The mathematician says, *Let's turn the pages*
and find the proof in the book of proofs.

He says, *It's as if it's already there*
somewhere just outside the door,

as if by sitting *zazen* in a coffee house,
someone could get through or get "across,"

or as if the theorems had already all
been written down on sheer

sheets of the invisible,
and held quite still,

so that to think hard enough
is simply to read and to recall—

the way the table remembers the tree,
the bowl remembers the kiln.

9. Concluding remarks

Everyone's mathematical education follows the path taken by our ancestors. It starts with counting in nursery rhymes—we learn about numbers by playing with rhyming words. Behind the game lies a serious intent: to teach counting by utilizing the power of poetry to engage learners' attention and enhance retention of abstract concepts. Poetry inspired by mathematics appears in the mathematics classroom through the ages, and at all mathematical levels. The nature and frequency of its use as a tool for teaching mathematics fluctuates to reflect technological advances and changing attitudes to mathematics education. But regardless of the specific reasons for the inclusion of a poem in a class, the power of poetry to engage attention and enhance memory is always an underlying presence. Some ancient cultures, for example India during the middle-ages, imparted most mathematical knowledge in verse form. Nowadays, poetry inspired by mathematics is used to shape course content by focusing attention on a particular aspect of the material taught in class, and acting as a springboard to initiate class-wide or small group discussions, assignments, or projects based on the poem's content. Judicious choice of poems and careful project construction often result in additional pedagogical benefits, such as better integration of material and easier transition to its applications. A different type of poetry project, with similar aims and results, requires students to compose their own poems about mathematical techniques or concepts. The nation wide education initiative 'writing across the curriculum' generated a number of recent pedagogical experiments with poetry writing in college mathematics classes. Examples of the use of poetry projects in college mathematics classes, a survey of the efforts made by educators in this direction and an extensive bibliography, may be found in [19,22]. A recent project in mathematics education that evaluates the effects of presenting advanced mathematical concepts to engineering students through a media-enhanced poetic interpretation may be found in [30].

The poems appearing in this article may be used in the mathematics classroom to enhance pedagogy or course content in any of the ways discussed above. They may also be used to enrich history of mathematics courses, and courses focusing on the connections between mathematics and the arts. Most of all, I hope that the poems presented here will inspire mathematicians and educators to write their own mathematical poems, and to experiment with innovative uses of poetry in their mathematics classes.

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